28/9: Lecture 1 - MOTIVATION & HISTORY

.svg / .svgz format
Stylus Labs.

Historically - Nala - Rituparna (links on Zulip).
Great Gambler - "Sakuni".

"Modern-day history" - Luca Pacioli (1494) - Problem of points (Wikipedia).
Game of rounds by two players. Equal chance of winning a round.
First player to win 'n' rounds is the winner. Say $n = 100$.
1^{st} player has won n_1 rounds & 2^{nd} player has won n_2 rounds. Game stops!
Prize Money $= P$. How does one split P?
Find prob. of Player 1 winning, say p_1. $p_2 = 1 - p_1$.
Player 1 gets $P \times p_1$ & Player 2 gets $P \times p_2$.
Pacioli proposed $p_1 = \frac{n_1}{n_1 + n_2}$; Niccolo Tartaglia in 1600's
Error! $n_1 = 2, n_2 = 0, n = 100 \Rightarrow p_1 = 1$
1560's Luiz Cardano wrote a book on "Games of chance" - not known until 1600's.

NT proposed p_1 based on $\frac{n_1 - n_2}{n}$

Error! $n = 100$; $n_1 = 75, n_2 = 65$
$n_1 = 99, n_2 = 89$ $\Longrightarrow$ p_1 based on $\frac{1}{10}$.

Resolved later by Fermat & Pascal.
beginning of prob. theory.
Gambler's ruin: (17th century) 2 players play. A winner of a round gets 1 INR, loser loses 1 INR.
Total money is n INR = (sum of money with both players)
Game ends when a player gets all n INR,
Players start with k_0 INR & $n - k_0$ INR.
They stop abruptly when they have "k" INR & "n-k" INR.
How to split the money? Pascal & Fermat solved it.

Laplace made a "prediction" of prob. that the sun will rise tomorrow.

Various attempts to formalize — von Mises (1900's) came close using a frequentist / empirical approach.

Still no axiomatic approach like Euclidean geometry.

[Ref: Wikipedia or Scheffe's book]

Kolmogorov laid down axiomatic framework in 1930's.

"DATA SCIENCE".

This course — "Intuitive / Heuristic / computational" approach.

Any mathematical discipline has

(1) Formal logical content. (basics in this course)

(2) Intuition & Ideas (lots in ")

(3) Applications (Good many)

Read Ch. 1 of Feller Vol. 1 for more on philosophy & motivation.

Caution: Unlike many math, prob. can be described in everyday english (toss a coin, roll a dice et al.) — advantageous but also scope for confusion.

Current-day applications:

Statistical physics: Exact positions of particles not fixed but random; spins in a magnet are random.

Biology: Genes mutate, evolve & combine randomly; Populations evolve randomly.

Data Science / Machine learning:

You go to Google — you type ISI, which ISI should google show?

"Predict under uncertainity"

Sally clark case;

Applications within mathematics –

– Probabilistic method (pioneered by Erdős).

Prob. ("object satisfies a constraint") > 0 $\Rightarrow$ $\exists$ one object satisfying the constraint

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– Two polynomials $P(x)$, $Q(x)$:

$P(x) = \sum_{i=0}^{n} c_i x^i$; $Q(x) = \prod_{i=1}^{n} (x-a_i) = \sum_{i=0}^{n} b_i x^i$

Is $P(x) = Q(x)$?

Take $\boxed{x_1, \cdots, x_{n+1}}$; suppose it is equal at $x_1, \cdots, x_5$ & not the rest. $\Rightarrow P \neq Q$.

suppose equal at $x_1, \cdots, x_{n+1}$ $\Rightarrow P = Q$.

Randomized algorithms – Choose x_1 "randomly" $\mathbb{P}(P(x_1) \neq Q(x_1))$.

Eg: Matrices A, B, C ($n \times n$ matrices) Is $AB = C$?

Casino's – Machines shuffle cards – Is the shuffling good?

Do long enough – Shuffling is good.

Information theory: n bits $x = 01 \cdots 1 \cdots 0 \cdots$ sent over a "noisy" network.

Bits are corrupted randomly. You receive $y = 10 \cdots 10$

Can we code "x" s.t. you can recover x from y?

" adding more bits