10/12/21

Comments on Quiz ques:

Part 2A

(Q1) In general:

- If there's a counterexample, then try one of:
  (a) letting one or more sets be empty or equal to S
  (b) Use simple 2-element sample spaces as an example.

Use these heuristics & seek to evaluate each of $\mathbb{P}(A\cap B)$, $\mathbb{P}(A)$, $\mathbb{P}(B)$.

(a) $\mathbb{P}(A\cap B)=0$, $\mathbb{P}(A)$, $\mathbb{P}(B)\neq 0$ is possible. F.

(b) $\mathbb{P}(A\cap B)=\mathbb{P}(A)\cdot|B|$: obviously false as RHS>1 can happen. F

(c) $\mathbb{P}(A\cup B)=\mathbb{P}(A)\cdot|B|$: same as above. F

(d) $\mathbb{P}(A\cap B)=\mathbb{P}(A)+\mathbb{P}(B)$ but this implies $\mathbb{P}(A\cap B)=\mathbb{P}(A\cap B)+\mathbb{P}(A\cup B)$ so $\mathbb{P}(A\cup B)=0 \Rightarrow \mathbb{P}(A)=\mathbb{P}(B)=0 \Rightarrow \mathbb{P}(A\cap B)=0 \Rightarrow$ independent; T

(e) The condition implies $\mathbb{P}(A\cap B)=0$, but $\mathbb{P}(A),\mathbb{P}(B)\neq 0$ can happen, F.

(f) F.

(Q2) $\mathbb{P}(O\mid E_1)=1$ obviously. For the other, here is why the answer is independent of p.

The point is, <u>ONCE</u> O_k occurs, this actually takes p out of the equation. Why? Because, once you know that exactly k routes are open, each subset of k routes becomes equally likely: so the prob. space, conditioned on O_k, is now uniform over the set of all subsets of size k.

Thus, the prob. that 1 belongs in that subset, is that 1 is one of k chosen elements, so k/n.

(Q3) An intelligent idea: use the idea of Y, N selection. Each subset is a confluence of N/Y choices:

	1	2	3	4	...	n	Set
A	Y	N	Y	N		N	$\{1,3\}$
B	N	N	N	Y		Y	$\{4,n\}$

We can choose two subsets using 2 sets of Y,N vectors. Now, observe that if $A\subseteq B$, then each red block <u>has</u> to be one of:

$\boxed{\begin{matrix}N\\N\end{matrix}}$ $\cancel{\boxed{\begin{matrix}Y\\N\end{matrix}}}$ $\boxed{\begin{matrix}Y\\Y\end{matrix}}$ $\boxed{\begin{matrix}N\\Y\end{matrix}}$

for each element of $\{1,2,\dots,n\}$. Conversely, n such blocks give $A\subseteq B$! So, for each element, 3 choices out of 4 i.e. $\left(\frac{3}{4}\right)^n$!

(Q4) Remember: if $\mathbb{P}(A)=1$ or $\mathbb{P}(A)=0$, then A occurs / doesn't occur <u>irrespective</u> of the occurrence / non-occurrence of other events. So, any probability zero/one event is always independent of any other event (including itself).

Parts b,d have counterexamples.

Part (b)

(Q1)

(a) $\mathbb{P}(A)=\frac{\lfloor \frac{n}{2}\rfloor}{n}$, $\mathbb{P}(B)=\frac{\lfloor \frac{n}{2}\rfloor}{n}$,
$\mathbb{P}(A\cap B)=\frac{\lfloor \frac{n}{2}\rfloor(\lfloor \frac{n}{2}\rfloor-1)}{n(n-1)} \Rightarrow \mathbb{P}(B\mid A)=\frac{\lfloor \frac{n}{2}\rfloor-1}{n-1}<\mathbb{P}(B)$.

(b) $\mathbb{P}(A)=\lfloor \frac{n}{2}\rfloor/n$, $\mathbb{P}(B)=\frac{n-\lfloor n/2\rfloor}{n}$

$\mathbb{P}(A\cap B)=\frac{\lfloor \frac{n}{2}\rfloor(n-\lfloor \frac{n}{2}\rfloor)}{n(n-1)} \Rightarrow \mathbb{P}(B\mid A)=\frac{n-\lfloor \frac{n}{2}\rfloor}{n-1}>\mathbb{P}(B)$.

(c) $\mathbb{P}(A)=4/n$, $\mathbb{P}(B)=n-6/n$
$\mathbb{P}(A\cap B)=4/n\cdot n-6/n-1 \Rightarrow \mathbb{P}(B\mid A)=\frac{n-6}{n-1}>\mathbb{P}(B)$.

I'll skip 2, it's explained well.

Ross - <u>Geometric</u> & <u>Negative</u> <u>Binomial</u>

To apply the theory of either the geometric <u>or</u> the negative binomial random variable, know:

- <u>What is a success, and what is a failure.</u>

Once you identify this, the problems are easy.
For example,

(8 a) As the experiment involves sampling with replacement, we see that the number of draws taken to wait for a ball which is black, is $Geom(M/M+N)$!
(a) will be $\mathbb{P}[Geom(M/M+N)=n]$.
(b) will be $\mathbb{P}[Geom(M/M+N)\geq k]$.

Things to practice for the geom. random variable:
(1) Geometric progression sum formula
(2) See ex 8(b) for computations involving <u>recursion</u>: often, while computing a quantity, it reproduces itself via a change of variable or a similarity transform.

<u>8(e)</u> The neg. binomial (p,k) is nothing but a sum of k independent Geom (p) random variables. It is important to think of it this way as well.
In the given qn., The "Banach match problem", we should ensure that we first divide the problem into 2 cases: one where the left pocket becomes empty, the other when the right becomes empty.
Once you fix, say, the former case, then taking matches from your left pocket is a success, and right is a failure. Thus, for there to be k matches in the other box, $NBin[1/2, N]=N+k$: use the formula to find $\mathbb{P}(NBin[1/2,N]=N+k)$.

→ Geom. random variables are used in 2 interesting situations.
(i) To model the power of a prime dividing a uniformly randomly chosen integer.
(ii) In epidemic modelling, infection spread between contacts is tracked via a geom. r.v.