21/22 oct

Agenda: • Problem of Points (Fermat)
- Qns of Assignment 2, 3
- Additional Problems
- Ross (?)

Problem of Points

$P_1\ P_2 \cdots P_n$ Q: How to divide prize money in this scenario?
$S_1\ S_2 \cdots S_n$ — no. of rounds away from winning the game.

each round, each player has $1/n$ chance of winning.

Set of outcomes

Two players: A B
$S_A = 1$ $S_B = 3$

You need to play at most 3 more rounds to get a winner.

$\Omega = \{AAA, AAB, ABA, ABB, \ldots, BBB\}$
The idea is that $\|\Omega\| = 8$.

A A B → winner of last round.
winner of third last ~~game~~ round
winner of second last round

Now, we have in all cases except BBB, that A wins the game.
So, $\mathbb{P}(A \text{ wins}) = 7/8$, $\mathbb{P}(B \text{ wins}) = 1/8$.
Thus, the money should be divided in the ratio 7:1.

Key idea: Construction & Thinking behind Sample Space.

Assignment 2

(Q10) Let A, N, P be the subsets of students that like analysis, n.t., prob respectively.
We can find

$$|A^c \cap N^c \cap P^c| = 50 - |A \cup N \cup P| \overset{\text{Incl. Excl.}}{=} 25$$

(a) $25/50 = 1/2$ (b) Sample space = Subsets of size 2
$\Rightarrow \binom{25}{2} / \binom{50}{2}$.

(Q3) $C = \{ck : c \text{ is a color}, k = 1, 2, \ldots, 10\}$.

$\Omega = \{S \subseteq C : |S| = 5\}$.

(a) Choose the color (b, bl, g, r, y), create events $E_b, E_{bl}, E_g, E_r, E_y$, note that:

$$E_b = \{bk_1 \ldots bk_5 : \{k_1 \ldots k_5\} \subseteq \{1, \ldots, 10\}\}$$

($k_1 \ldots k_5$ distinct) and finish.

(b) Choose the no. appearing 3 times, choose the no. appearing 2 times, then choose the colours, so:

$$\frac{\boxed{10 \times 9} \times \binom{5}{3} \times \binom{5}{2}}{\binom{50}{5}}$$

remember, which no. is chosen for which purpose, matters here.

(Q5) Can place rooks conditionally:

R_1	–	–	–
–	–	R_3	–
–	R_2	–	–
–	–	–	R_4

Rook 1 placed in 16 ways, then rook 2 in 9 ways ... leads to $16 \times 9 \times 4 \times 1$, but don't forget to divide by orderings of rooks i.e. $4!$, so the answer is just $4!$.

So $8! / \binom{64}{8}$.

(Q8) The assumption that $\mathbb{P}(\{\omega_1, \omega_2\}) = 1/|\Omega_1 \times \Omega_2|$ was not to be assumed. (It is an independence assumption)

The notion of a pr. space goes beyond independence. E.g.

$\Omega_1 = \Omega_2 = \{0, 1\}$

$P(0,0) = 1/6$, $P(0,1) = 1/3$.
$P(1,0) = 1/4$, $P(1,1) = 1/4$.

Then $(\Omega_1 \times \Omega_2, P)$ is a pr. space. If we look at it as 2 coins being tossed, then:
The 2nd coin depends on the 1st coin.
Hence, we can see why the independence condition fails to hold.

Additional Remarks

- In the rooks problem, we can create a bijection between the set of all non-attacking rook posns & the set of all $f : \{1, \ldots, 8\} \circlearrowleft$ bijective.

  By placing rooks in $(i, f(i))^{th}$ posns. $i = 1, 2, \ldots, 8$.

- The cartesian prod. is used to combine different experiments into one.

- Q10 can be done via a Venn diagram as well.

- For useful math content:
  - tricki.org
  - Brilliant.org
  - Art of Problem Solving (AoPS).