24/09/2021 and 1/10/2021

Saturday, 2 October 2021 6:17 PM

24/09

"where there is randomization, there is probability"

Probability lends itself to mathematical use, because in situations where any experiment (mathematical or not) is being performed, one would like to weigh the outcomes of the experiment by their likelihood of occuring.

"If you hate probability: explore the connections between probability and other parts of mathematics, and see how probability can provide new insights"

Examples:

— Given a prime p, how "likely" is it that a given number is divisible by p?

You expect this to be $\boxed{1/p}$, right?

Insight: For two different primes $p \neq q$, the information that a number is divisible by p, DOES NOT affect the likelihood of it being divisible by q.

E.g. The "likelihood" that a randomly chosen number is even, is $1/2$.

→ The likelihood that a randomly chosen number is even, given that it is a multiple of 3, is still $1/2$!

This phenomena is referred to as independence of events. Independence is one of the most important concepts in probability.

Independence

→ Another common phenomena in probability, is that of Concentration: Concentration is the idea that in an experiment, a small number of outcomes may have a large probability of occuring.

E.g: Roll 2 six-sided dice & sum the numbers on top. Even though there are 11 outcomes (2, 3, 4 ... 11, 12), three of them i.e. {6, 7, 8} have a collective prob of > 1/3 of occurring.

Probabilistic Principle: Independent events submit themselves to concentration phenomena.

E.g: Erdos-Kac formula for number of prime factors of randomly chosen number: for large n, n has ≈ log log n prime factors.

General Tips

— math.stackexchange.com: Repository for mathematics Q & A.
→ If used properly, can save > 1 hr of study time per day.
→ Please don't copy from here!

— scihub.ren : For accessing math papers. Insert DOI into the search box, and 95% of the time your paper is available.

— libgen.rs / gen.lib.rus.ec: For textbooks, use author & title to search. Download DJVU reader if necessary, to read DJVU books.

Other Tips

→ Don't study too much, but study efficiently. I would say: on days of low motivation, consider < 2 hrs. of study. It's all about energy levels, eat well, sleep well, recharge well.

→ Talk: Friends, family, counsellor etc., but social health is important.

1/10/21

Principle: Understand what is fixed and what is random, to create your sample space.

Assignment clarification:

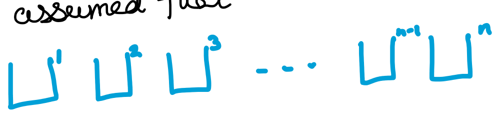
→ If necessary, please use unions ($\cup$) & intersections ($\cap$) to describe your set. Qualifiers such as $\wedge, \vee, \neg$ (not) may also be used. It is not necessary to specify the cardinality of your sample space.

Q2: The notation used in the sample space description is like:

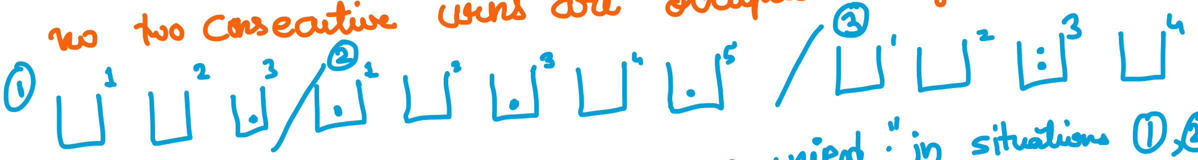
$$(x_1, x_2, \dots, x_n)$$

- x_1: no. of objects in 1^{st} box
- x_2: no. of objects in 2^{nd} box
- x_n: no. of objects in n^{th} box.

It is usually assumed that the boxes are kept side-to-side, like



Consecutive means "two boxes next to each other". Thus, in all questions, you need to describe the elements of the sample space where no two consecutive urns are occupied. To give examples:



"No two consecutive urns are occupied" in situations ①, ②, ③.

Q4: What is random: the binary string of length n i.e. your input.

What is fixed: What the encoder does with the string k

Eg.: $n=k=3$: You receive Input: 101 | 011

Encoder $\downarrow$ $k=3$ | Encoder $\downarrow$ $k=3$

Encoded Output: 111 000 111 | 000 111 111

Sample space consists of all possible encoded outputs.

Q5(b): · Every element of the sample space, is a set.

· Every element of that set, is a pair of people.

e.g. if A is a friend of B & C is a friend of D, then {(A,B), (C,D)} would be an element of the samp. space.

→ It is possible that nobody be friends with each other, or that everybody are friends with each other. Key point: Your sample space should be set up to capture all possible relations between the people.

(Side note: people can't be friends with themselves)

Combinatorics:

Ross Ch 1 Self-Test:

① The key idea is that there is a symmetry between arrangements of one kind vs arrangements of the other kinds. E.g.

ABCDEF ⟷ BACDEF

DACEFB ⟷ DBCEFA

CDFABE ⟷ CDFBAE

"A comes before B".
"B comes before A".

This allows us to get our answer by dividing the sample space using these symmetries. For (d) the answer is $\frac{6!}{2! \times 2!}$,

For (c), we have:

DAEFBC DBEFAC DAEFCB

DCEFAB DCEFBA DBEFCA

and thus the answer is $\frac{6!}{3!}$.

Another way: For (d), by choice.

→ We choose 2 positions for A & B. This is $\binom{6}{2}$.

→ Of the remaining 4, we choose 2 positions for C&D. This is $\binom{4}{2}$.

→ We can put E & F in any order in the last 2, so:

$$\binom{6}{2} \times \binom{4}{2} \times 2.$$

Both answers are correct.

Avoid Infinite Unions & Sums until covered in class: there is some counter-intuitive reasoning involved.