Probability I: Quiz 1

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MAXIMUM MARKS : 15   Time : 75 mts.

ALL QUESTIONS CARRY 7.5 POINTS. ATTEMPT ANY TWO OF THEM ONLY.

Submit solutions via Moodle by 5.00 PM on November 9th.

Please write your name and the honesty statement below on your answer script and sign below the same. Else 3 points will be deducted.

I have not received, I have not given, nor will I give or receive, any assistance to another student taking this exam, including discussing the exam with other students. The solution to the problems are my own and I have not copied it from anywhere else. I have used only class notes, notes from TA sessions, my own notes and assignment solutions.

Define sample space, pmf or PD and events properly before computing anything.

1. There are n different routes from ISI to the Bangalore airport. BBMP (Bangalore Municipal authority) choses uniformly at random a subset S of the n routes and closes them for repair. Routes not closed for repair are said to be open.

   (a) Unaware of the closed routes, you choose a route at random. What is the probability that you have chosen an open route ? **(3.5)**

   (b) Not trusting your choice, your friend a chooses an alternate route (different from your route) at random. What is the probability that one of you will have chosen an open route ? **(4)**

2. Let $(\Omega, \mathbb{P})$ be a probability space.

   (a) Suppose $a_1, \ldots, a_n$ are arbitrary non-negative numbers and $A_1, \ldots, A_n$ be pairwise disjoint sets such that $\Omega = \cup_{i=1}^n A_i$. Assume that $\sum_{i=1}^n a_i \mathbb{P}(A_i) > 0$. Show that Q defined as follows is also a probability distribution on Ω: **(5)**

   $$Q(A) = \frac{\sum_{i=1}^n a_i \mathbb{P}(A \cap A_i)}{\sum_{i=1}^n a_i \mathbb{P}(A_i)}.$$

   (b) Suppose $A_1, \ldots, A_n$ be pairwise disjoint sets with $\Omega = \cup_{i=1}^n A_i$ but a_i's are real numbers (not necessarily non-negative) and $\sum_{i=1}^n a_i \mathbb{P}(A_i) > 0$. Is Q still a probability distribution ? **(2.5)**

   If you are claiming that Q is not a probability distribution, please show explicitly which of the conditions is violated and why.

3. Suppose a random sequence of n coin tosses (i.e., an element of $\{0,1\}^n$) is chosen according to product Bernoulli distribution with parameter p. Compute the following probabilities.

   (a) Let $a = (a_1, \ldots, a_n) \in \{0,1\}^n$ be fixed sequence of coin tosses. What is the probability that the random sequence of n coin tosses matches with the sequence a at exactly k indices ? **(3.5)**

   (b) What is the probability that there are no successive heads in the first three tosses in the random sequence of coin tosses ? Successive heads in the first three tosses means the first two tosses are heads or the next two (second and third) tosses are heads. **(4)**

4. An urn initially contains n red and m blue balls. In the below experiments, randomly refers to uniformly at random.

   (a) In the first experiment, the balls are withdrawn randomly one at a time until a total of r ($r \leq n$) red balls have been withdrawn. What is the probability that a total of k balls are withdrawn? **(5)**

(b) In the second experiment, they are withdrawn randomly one at a time until all the balls have been withdrawn. What is the probability that all the red balls are withdrawn before all the blue ones have been withdrawn? **(2.5)**