Probability I: Quiz 2

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MAXIMUM MARKS : 7 Time : 60 mts.

ALL QUESTIONS CARRY 7 POINTS. ATTEMPT ANY ONE ONLY.

Submit solutions via Moodle by 7.10 PM on December 9th.

Please write your name and the honesty statement below on your answer script and sign below the same. Else 3 points will be deducted.

I have not received, I have not given, nor will I give or receive, any assistance to another student taking this exam, including discussing the exam with other students. The solution to the problems are my own and I have not copied it from anywhere else. I have used only class notes, notes from TA sessions, my own notes and assignment solutions.

Define sample space, pmf/PD, events and random variables properly before computing anything.

1. A box contains n coupons labelled $1, 2, \ldots, n$. Coupons are drawn one after another without replacement (a coupon is drawn, the number noted and discarded, the next coupon is drawn, etc). In each of the following cases, prove or disprove that A and B are independent and also compute $\mathbb{P}(B|A)$ and $\mathbb{P}(B)$.
   - (a) Let A be the event that the first coupon drawn is an even number. Let B be the event that the second coupon drawn is an even number. **(2)**
   - (b) Let A be the event that the first coupon drawn is an even number. Let B be the event that the second coupon drawn is an odd number. **(2)**
   - (c) Let A be the event that the first coupon draw is at most 4 and B be the event that the second coupon drawn is at least 7. **(3)**
2. A drunkard has n keys in his pocket and only one of the keys is the correct one to open his house. For two successive nights, he has forgotten the correct key to open his house. Compute the expected number of attempts in the following two scenarios :
   - (a) SAMPLING WITHOUT REPLACEMENT:
     On the first night, he tries the keys from his pocket at random one by one but drops down the unsuccessful keys. Let N stand for the random variable denoting the number of attempts (including the successful one)to open the door. Compute $\mathbb{E}(N)$.
   - (b) SAMPLING WITH REPLACEMENT:
     On the second night, he still tries keys from his pocket at random but he is so drunk that he puts back the unsuccessful keys again in his pocket. So, it is possible that he can select the same key multiple times. If he does not succeed in n attempts, he gives up. Here, let N stand for the random variable denoting the number of unsuccessful attempts (in the case he has given up, the number of unsuccessful attempts is to be taken as n). Compute $\mathbb{E}(N)$.
3. Compute the pmf and mean of X in the following two examples.
   - (a) Consider the Bose-Einstein probability distribution with r balls and n urns. Let X be the number of empty urns. **(4)**
   - (b) Consider the Maxwell-Boltzmann probability distribution with r balls and n urns. Let X be the number of empty urns. **(3)**