Indian Statistical Institute

HW - 3: Analysis of Several Variables. Due date: 17/10/2022 Instructor : Jaydeb Sarkar

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NOTE: $\mathcal{O}_n \subseteq \mathbb{R}^n$ open subset.

(1) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$f(x,y) = (x^2y, xy^2) \qquad ((x,y) \in \mathbb{R}^2).$$

Prove that $f$ is locally invertible around all $(x,y) \in \mathbb{R}^2$ such that $x \neq 0$ and $y \neq 0$. Compute the differential of the local inverse of $f$ at the point $f(2,1)$.

(2) Prove that the $xy + 2^x - 2^y = 0$ defines a differentiable implicit function $y = f(x)$ around $(0,0)$. Compute $f'(0)$ and $f''(0)$.

(3) Find the points where the function

$$f(x,y) = (\sin x \cosh y, \cos x \sinh y) \qquad (x, y \in \mathbb{R}).$$

has a local inverse.

(4) Prove that the function

$$f(x,y) = (e^x \cos y, e^x \sin y) \qquad (x, y \in \mathbb{R}),$$

has a local inverse at each point, but it does not have a global inverse.

(5) Consider the system of equations

$$3x + y - z - w^3 = 0,\ x - y + 2z + w = 0,\ 2x + 2y - 3z + 2w = 0,$$

can be solved for $x, y, w$ in terms of $z$ but not for $x, y, z$ in terms of $w$.

(6) Suppose the equation $f(x,y,z) = 0$, where $f$ is differentiable, can be solved for each of the three variables $x, y, z$ as a differentiable function of the other two. Show that at any point where $f(x,y,z) = 0$ and at least one of the partial derivatives $f_x, f_y, f_z$ is nonzero, the other two are also nonzero, and

$$\frac{\partial x}{\partial y}\frac{\partial y}{\partial z}\frac{\partial z}{\partial x} = -1.$$