Indian Statistical Institute

HW - 6: Analysis of Several Variables. Due date: 11/11/2022 Instructor : Jaydeb Sarkar

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(1) Evaluate the line integral of the function $f(x,y,z) = xy + y + z$ along the curve $\gamma(t) = \langle 2t, t, 2-2t\rangle$, $t \in [0,1]$.

(2) Evaluate the line integral of $f(x,y,z) = \sqrt{x^2+z^2}$ along the curve $\gamma(t) = \langle 0, \cos t, \sin t\rangle$, $t \in [0, \frac{\pi}{2}]$.

(3) Compute $\int_C (x + \sqrt{y} - z^2)\, ds$, where $C = C_1 \cup C_2$, and parametrizations of C_1 and C_2 are given by $\gamma_1(t) = \langle t, t^2, 0\rangle$ and $\gamma_1(t) = \langle 1, 1, t\rangle$, $t \in [0,1]$, respectively.

(4) Find the work done by the force field $F(x,y,z) = \langle -\frac{1}{2}x, -\frac{1}{2}y, -\frac{1}{4}\rangle$ on a particle as it moves along the helix $\gamma(t) = \langle \cos t, \sin t, t\rangle$ from $(1,0,0)$ to $(-1,0,3\pi)$.

(5) Evaluate the line integral $\int_C x^2 y\, dx + (x-2y)\, dy$, where C is the part of the parabola $y = x^2$ from $(0,0)$ to $(1,1)$.

[Remark: Let $F = \langle f, g, h\rangle$ be a vector field, and let $\gamma(t) = \langle x(t), y(t), z(t)\rangle = x(t)i + y(t)j + z(t)k$, $t \in [a,b]$, be a parametrization of C. We know that $\int_C F \cdot dr = \int_a^b F(\gamma(t)) \cdot \gamma'(t) dt$. However, note that

$$\int_a^b F(\gamma(t)) \cdot \gamma'(t)\, dt = \int_a^b \langle f(\gamma(t)), g(\gamma(t)), h(\gamma(t))\rangle \cdot \langle dx/dt, dy/dt, dz/dt\rangle\, dt,$$

and so $\int_C F \cdot dr = \int_C f dx + g dy + h dz$.]

(6) Let C be the line segment joining the points (x_1, y_1) to (x_2, y_2). Prove that

$$\int_C x\, dy - y\, dx = \det \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix}.$$

(7) Find a parametric representation for the curve resulting by the intersection of the plane $3x + y + z = 1$ and the cylinder $x^2 + 2y^2 = 1$ in $\mathbb{R}^3$.

(8) Find a parametrization for the solid triangle $\Delta := \Delta ABC$, where $A = (1,0,0)$, $B = (0,1,0)$, and $C = (0,0,1)$. Also find a parametrization for the curve $\partial\Delta$.

(9) Compute the equation of the surface of revolution generated by revolving the hyperbola $x^2 - 4z^2 = 1$ about the z-axis.

(10) Compute the area of the piece of the paraboloid $z = x^2 + y^2$ which is cut out by the region between the cylinder $x^2 + y^2 = 2$ and the cylinder $x^2 + y^2 = 6$.

(11) Find a unit vector normal to the surface $z = 4 - x^3 - y^3$ at the point $(1,1,2)$. Also compute the tangent place to the surface at $(1,1,2)$.

(12) Find the equation of the tangent plane to the surface $z = x\exp(-2y)$ at the point $(1,0,1)$.

(13) Let $a > 0$. Find the equation of the tangent plane to the torus

$$r(u,v) = ((a + \sin v)\cos u, (a + \sin v)\sin u, \cos v),$$

at $r(0,0)$.