(1) Find the area of the surface $\langle v\cos u, v\sin u, u\rangle$, for $0 \le u \le \pi$ and $0 \le v \le 1$.

(2) Find the area of (i) the portion of $2x+4y+z=0$ inside $x^2+y^2=1$; (ii) $z=\sqrt{x^2+y^2}$ that lies below $z=2$; (iii) the cone $z=k\sqrt{x^2+y^2}$, $k>0$, that lies over a region R with area a.

(3) An object moves from $(1,0,0)$ to $(-1,0,\pi)$ along the curve $r(t)=\langle \cos t, \sin t, t\rangle$, subject to the force field $\langle y^2, y^2, xz\rangle$. Find the work done.

(4) Evaluate the surface integral $\int_S xy\, dS$, where S is the triangular region with vertices $(1,0,0)$, $(0,2,0)$ and $(0,0,2)$. [Ans: $\frac{1}{\sqrt{6}}$. Here the region of parametrization is given by the plane $z=2-2x-y$.]

(5) Let F and G be two C^1-vector fields on $\mathbb{R}^3$. Prove that

$$div(F\times G)=curl(F)\cdot G+F\cdot curl(G).$$

(6) Use Green's theorem to prove that

$$\int_C -x^2y\,dx+xy^2\,dy=8\pi,$$

where C is the circle $x^2+y^2=4$ (with counterclockwise orientation).

(7) Recall that the area of a (suitable) region R is given by

$$\text{area}(R)=\frac{1}{2}\int_C \langle -y, x\rangle\cdot dr,$$

where C is the boundary of R. Explain this relation geometrically.

(8) For the field $F=\langle x,y,z\rangle$ and the cylinder V given by $x^2+y^2\le 1$, $0\le z\le 1$;
   (i) Compute $\nabla\cdot F$ and $\int_V \nabla\cdot F$.
   (ii) Compute $\int_{\partial V} F\cdot dS$ (outward normal direction). Is this supposed to match (i)?

(9) Determine whether or not $F=\langle z\sec^2 x, z, y+\tan x\rangle$ is conservative. If F is conservative, find a potential function for F.

(10) For the gravitational field $F(x)=-k\frac{x}{\|x\|}$ in $\mathbb{R}^3$ and the sphere $V=\{x\in\mathbb{R}^3 : \|x\|\le 1\}$;
   (i) Compute $\nabla\cdot F$ and $\int_V \nabla\cdot F$.
   (ii) Compute $\int_{\partial V} F\cdot dS$ (outward normal direction). Is this supposed to match (i)?

(11) Find the flux of $F(x,y,z)=(x,y,3)$ out of the region bounded by the paraboloid $z=x^2+y^2$ and the plane $z=4$.

(12) (i) Prove that curl $\circ\, \nabla = 0$ and div $\circ$ curl $=0$. (ii) Prove that the curl through all closed surfaces in $\mathbb{R}^3$ has zero flux. That is, if S is a closed surface in the domain of a field F, then

$$\int_S (\nabla\times F)\cdot dS=0.$$

(13) Explain why the surface $z=x^2\cos y$ is orientable.

(14) If $R=\{(x,y) : |x|\le 2, |y|\le 1\}$, then compute $\int_{\partial R} e^{2x+3y}\,dx+e^{xy}\,dy$.

(15) If $R=\{(x,y) : |x|\le 1, x^2\le y\le 1\}$, then compute $\int_{\partial R}\sqrt{1+x^2}\,dy$.

(16) Let $F(x,y,z)=\langle y, z, -2x\rangle$, and let C be a simple closed curve contained in the plane $x+y+z=1$. Prove that $\int_C F\cdot dr=0$.

(17) Verify Stokes's theorem for the field $F = -yi + xj + e^z \ln(1+z)k$ and the surface $z = \sqrt{4 - x^2 + y^2}$. (Find separately the path integral around the edge and the flux of the curl. You must decide which sense of the normal to pick.)

(18) Compute the outward flux $\int_S F \cdot dS$, where S is the sphere $x^2 + y^2 + z^2 = 4$ and $F(x, y, z) = (x^2 + y^2 + z^2)(xi + yj + zk)$. [Hint: Divergence theorem]