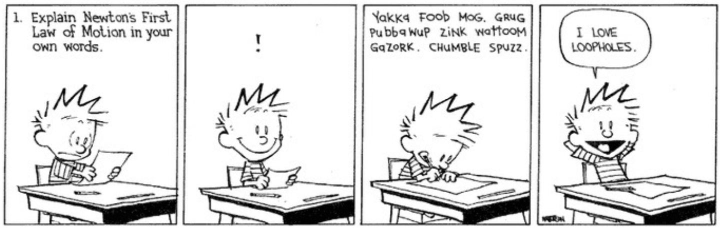
Physics I
ISI B.Math
HW set 1



1. A bee flies on a trajectory such that its polar coordinates at time t are given by $r = \frac{bt}{\tau^2}(2\tau - t), \theta = \frac{t}{\tau}$, $(0 \leq t \leq 2\tau)$, where b and τ are positive constants. Find the velocity vector of the bee at time t. Show that the least speed achieved by the bee is $\frac{b}{\tau}$. Find the acceleration of the bee at this instant.

2. The luckless Daniel (D) is thrown into a circular arena of radius a containing a lion (L). Initially the lion is at the centre (O) of the arena while Daniel is at the perimeter. Daniel's strategy is to run with his maximum speed u around the perimeter. The lion responds by running at his maximum speed U is such a way that it remains on the (moving) radius OD. Show that r, the distance of L from O, satisfies the differential equation

$$\dot{r}^2 = \frac{u^2}{r^2}\left(\frac{U^2 a^2}{u^2} - r^2\right)$$

find r as a function of t. If $U \geq u$, show that Daniel will be caught, and find out how long this will take. Show that the path taken by the lion is an arc of a circle. For the special case in which $u = U$ sketch the path taken by the lion and find the point of capture.

3. Consider a particle of mass m whose motion starts from rest in a constant gravitational field. If a resisting force proportional to the square of the velocity (i.e, kmv^2) is encountered, show that the distance s the particle falls in accelerating from v_0 to v_1 is given by

$$s(v_0 \to v_1) = \frac{1}{2k} \ln\left[\frac{g - kv_0^2}{g - kv_1^2}\right]$$

4. A beach ball is thrown upward with initial speed v_0. Assume that the drag force from the air is $F_d = -m\alpha v$. Show that the speed of the ball v_f, right before it hits the ground is given by the implicit equation

$$v_0 + v_f = \frac{g}{\alpha} \ln\left(\frac{g + \alpha v_0}{g - \alpha v_f}\right)$$

Does the ball spend more time or less time in the air than it would if it were thrown in vacuum ?

5. A particle is projected vertically upward in a constant gravitational field with an initial speed v_0.Show that if there is a retarding force proportional to the square of the instantaneous speed, the speed of the particle when it returns to the initial position is

$$\frac{v_0 v_t}{\sqrt{v_0^2 + v_t^2}}$$

where v_t is the terminal speed.

6. The force on a particle of charge q and mass m moving with a velocity $\mathbf{v}$ in an electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ is given by

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

(a) Choose the z-axis to lie in the direction of $\mathbf{B}$ and let the plane containing $\mathbf{E}$ and $\mathbf{B}$ be the yz plane. Thus $\mathbf{B} = B\mathbf{k}$ and $\mathbf{E} = E_y\mathbf{j} + E_z\mathbf{k}$

Show that the z component of the motion is given by

$$z(t) = z_0 + \dot{z}_0 t + \frac{qE_z}{2m}t^2$$

where $z(0) = z_0$ and $\dot{z}(0) = \dot{z}_0$

(c) Continue the calculation and obtain expressions for $\dot{x}(t)$ and $\dot{y}(t)$. Show that the time averages of these velocity components are

$$< \dot{x}(t) >= \frac{E_y}{B}, < \dot{y}(t) >= 0$$

(d) Integrate the velocity equations found in (c) and show , (with the initial conditions $x(0) = -\frac{A}{\omega_c}, \dot{x}(0) = \frac{E_y}{B}, y(0) = 0, \dot{y}(0) = A, \omega_c = \frac{qB}{m}$) that

$$x(t) = \frac{-A}{\omega_c}\cos\omega_c t + \frac{E_y}{B}t, y(t) = \frac{A}{\omega_c}\sin\omega_c t$$

These are parametric equations of a trochoid.