Physics I
ISI B.Math
HW set 2

1. A particle P of unit mass moves on the positive x- axis under the force field

$$F = \frac{36}{x^3} - \frac{9}{x^2}$$

where $x > 0$.
Show that the motion of P consists of either(i) periodic oscillation between two extreme points or (ii) an unbounded motion with one extreme point, depending upon the value of total energy. Initially, P is projected from the point $x = 4$ with speed 0.5. Show that P oscillates between two extreme points and find the period of the motion. You may make use of the formula

$$\int_a^b \frac{xdx}{[(x-a)(b-x)]^{\frac{1}{2}}} = \frac{\pi(a+b)}{2}$$

Show that there is a single equilibrium position for P and that it is stable. Find the period of small oscillations about this point.

2. A particle is under the influence of a force $F = -kx + \frac{kx^3}{a^2}$, where k and α are constants and k is positive. Determine $U(x)$ and discuss the motion. What happens when the total energy $E = \frac{1}{4}k\alpha^2$?

3. A particle moves towards $x = 0$ under the influence of a potential $V(x) = -A|x|^n$, where $A > 0$ and $n > 0$. The particle has barely enough energy to reach $x = 0$. For what values of n will it reach $x = 0$ in finite time?

4. Which of the following forces are conservative ? If conservative, find the potential energy $U(\mathbf{r})$.
(a) $F_x = ayz + bx + c, F_y = axz + bz, F_z = axy + by$
(b) $F_x = -ze^{-x}, F_y = \ln z, F_z = e^{-x} + \frac{y}{z}$

5. an overdamped harmonic oscillator satisfies the equation

$$\ddot{x} + 10\dot{x} + 16x = 0$$

At time $t = 0$, the particle is projected from the point $x = 1$ toward the origin with speed u. Find $x(t)$. Show that the particle will reach the origin at some later time t if

$$\frac{u-2}{u-8} = e^{6t}$$

How large must u be so that the particle will pass through the origin?

6. A mass on the end of a spring (with natural frequency ω_0) is released from rest at position x_0. The experiment is repeated, but now with the system immersed in a fluid that causes the motion to be overdamped (with damping coefficient β). Find the ratio of the maximum speed in the former case to that in the latter. What is the ratio in the limit of strong damping ($\beta \gg \omega_0$) ? In the limit of critical damping?

7. A child of mass m sits in a swing of negligible mass suspended by a rope of length l. Assume that the dimensions of the child are negligible compared to l. His father pulls him back until the

rope makes an angle of 1 radian with the vertical, then pushes with a force $F = mg$ along the arc of a circle of radius l until the rope is vertical, and releases the swing. For what duration of time did the father push the swing ? You may assume $\sin\theta \approx \theta$ for $\theta < 1$

8. As the damping on an oscillator is increased there comes a point when the name "oscillator" seems barely appropriate. (a) To illustrate this, prove that a critically damped oscillator can never pass through the origin $x = 0$ more than once (b) Prove the same for an overdamped oscillator.