Physics I
ISI B.Math
HW set 3

1. Let $f(y)$ be a given function of y. Then show that the function $y(x)$ that extremizes the integral

$$\int_{x_1}^{x_2} f(y)\sqrt{1+y'^2}dx$$

satisfies the differential equation $1 + y'^2 = Bf^2(y)$ where B is a constant.

2. Assume that the speed of light in a given slab of material is proportional to the height above the base of the slab. Show that light moves in a circular arc in this material. You may assume that light takes the path of least time between two points (Fermat's principle).[Hint: You could use the result derived in 1.]

3. A particle moves in a plane under the influence of a force $f = -Ar^{\alpha-1}$ directed towards the origin; A and $\alpha(> 0)$ are constants. Choose appropriate generalized coordinates and let the potential energy be zero at the origin. Find the Lagrangian equations of motion. Is the angular momentum about the origin conserved ? Is the total energy conserved ?

4. A double pendulum consists of two simple pendula, with one pendulum suspended from the bob of the other. If the two pendula have equal lengths and have bobs of equal mass and are confined to the same plane, find Lagrange's equations of motion for the system. Do not assume small angles.

5. Find the equations of motion for an "elastic pendulum": A particle of mass m is attached to an elastic string of stiffness K and unstretched length l_0. The spring is arranged to lie in a straight line (which we can arrange by, say, wrapping the spring around a massless rod. Assume that the mass moves in a vertical plane. Solve the Lagrange equations in the approximation of small angular and radial displacements from equilibrium and corresponding small velocities.

6. A simple pendulum of length b and a bob with mass m is attached to a massless support moving vertically upward with constant acceleration a. Determine (a) the equations of motion and (b) the period for small oscillations.

7. A mass M is free to slide along a frictionless rail. A pendulum of length l and mass m hangs from M. Find the equations of motion and the general solutions in a small angle approximation.

8. A particle of mass m rests on a smooth plane. The plane is raised to an inclination angle θ at a constant rate $\alpha(\theta = 0$ at $t = 0)$, causing the particle to move down the plane. Determine the motion of the particle.

9. A particle of mass m moves in one dimension under the influence of a force

$$F(x,t) = \frac{k}{x^2}e^{-\frac{t}{\tau}}$$

where t and τ are positive constants . Compute the Lagrangian and the Hamiltonian functions. Compare the Hamiltonian and the total energy, and discuss the conservation of energy for the system.

10. A particle of mass m slides on the inside surface of a frictionless cone. The cone is fixed with its tip on the ground and its axis vertical. The half angle at the tip is α. Let r be the distance from the particle to the axis and let θ be the angle around the cone. Find the equations of motion.
If the particle moves in a circle of radius r_0, what is the angular frequency ω of this motion ?
If the particle is then perturbed slightly from this circular motion, what is the frequency Ω of oscillations about the radius r_0 ? Under what conditions will $\Omega = \omega$?