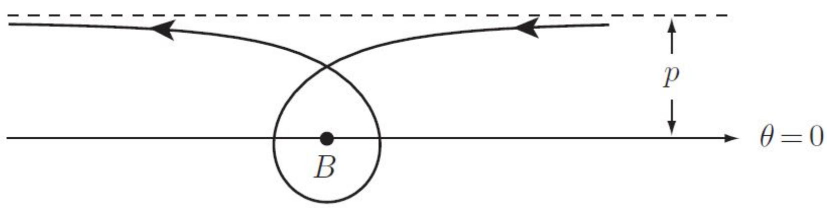
Physics I
ISI B.Math
HW set 4

1. A particle moves in circular orbit in a force field given by $\mathbf{F}(\mathbf{r}) = -\frac{k}{r^2}\hat{\mathbf{r}}$. Show that if k suddenly decreases to half its original value, the particle's orbit becomes parabolic.

2. Find the central force law that allows a particle to move in a spiral orbit given by $r = k\theta^2$ where k is a constant.

3. Two particles moving under the influence of their mutual gravitational force describe circular orbits about one another with a period τ. If they are suddenly stopped in their orbits and allowed to gravitate towards each other, show that they will collide after a time $\frac{\tau}{4\sqrt{2}}$.

4. A particle moves under the influence of a central force given by $F(r) = -\frac{k}{r^n}$. If the particle's orbit is circular and passes through the centre of the force. show that $n = 5$.



5. The engine of a spaceship have failed and the ship is moving in a straight line with speed V. The crew calculate that their present course will miss the planet B-Zar by a distance p as shown in the above figure. However, B-Zar is known to exert the force $\mathbf{F} = -\frac{m\gamma}{r^3}\hat{\mathbf{r}}$ on any mass in the vicinity. A measurement of the constant γ reveals that $\gamma = \frac{8p^2V^2}{9}$. Show that the crew of the spaceship will get a free tour around the planet before continuing on their original path. What is the distance of closest approach and what is the speed of the spaceship at that instant ? [Hint: Use the path equation.]

6. Suppose that the sun were surrounded by a dust cloud of uniform density ρ which extended at least as far as the orbital radius of the earth. The effect of the dust cloud is to modify the gravitational force experienced by the earth, so that the potential energy of the earth is

$$U(r) = -\frac{GmM}{r} + \frac{1}{2}kr^2$$

where M is the mass of the sun, m the mass of the earth G the gravitational constant and $k = \frac{4\pi\rho mG}{3}$. Note that $k > 0$, so that this additional term is attractive. The effect of the dust cloud is to cause the elliptical orbits about the sun to precess slowly.

(a) Find the force $\mathbf{F}$ acting on the earth.

(b) Make a careful sketch of the effective potential $U_{eff}(r)$. On your sketch, indicate (i) the energy E_0 and the radius r_0 of a circular orbit, and (ii) the energy E_1 and turning points r_1 and r_2 of an orbit that is not circular.

(c) Assume the earth is in a circular orbit of radius r_0 about the sun. Derive the equation satisfied by r_0 in terms of the angular momentum l, and the constants m, M, G and k. You need not solve the equation.

(d) Show that the frequency of small oscillations ω_r about the circular orbit of radius r_0 can be written as

$$\omega_r = \sqrt{{\omega_0}^2 + \frac{3k}{m}}$$

where ω_0 is the angular velocity of revolution about the sun for a circular orbit.

(e) Finally , assuming that k is small, show that the precession frequency $\omega_p = \omega_r - \omega_0 = \frac{3k}{2m\omega_0}$

7. A single particle of mass m, momentum $\mathbf{p}$ and angular momentum $\mathbf{L}$ about the center of force is acted on by an inverse square central force described by $\mathbf{F}(\mathbf{r}) = -\frac{k}{r^2}\hat{\mathbf{r}}$

(a) Show that, the Runge-Lenz-Laplace vector, given by

$$\mathbf{A} = \mathbf{p}\times\mathbf{L} - mk\hat{\mathbf{r}} \tag{1}$$

is conserved during this motion.

(b) In class we have shown that under the above inverse square force , when the total energy $E < 0$, the trajectory of the particle is an ellipse, with the centre of force at one focus of the ellipse. Given this situation, show that the Lenz vector points along the semi-major axis of the ellipse away from the centre.[Hint: Find out the direction of the Lenz vector at the point on the orbit closest to the focus]

(c) Show that the momentum vector moves in a circle of radius $\frac{mk}{L}$ centred on $(0, A/L)$, i.e,

$$p_x^2 + (p_y - A/L)^2 = (mk/L)^2$$

where the x-axis is is chosen along the semi major axis and the z-axis is chosen along L. [Hint: Use an appropriate squared form of equation]