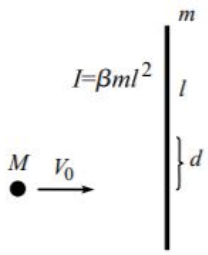
Physics I
ISI B.Math
HW set 5

1. Consider a thin homogeneous plate that lies in the $x_1 - x_2$ plane.
Show that the inertia takes the form

$$\{\mathbf{I}\} = \begin{Bmatrix} A & -C & 0 \\ -C & B & 0 \\ 0 & 0 & A+B \end{Bmatrix}$$

2. A uniform ball of mass M and radius a is pivoted so that it can turn freely about one of its diameters which is fixed in a vertical position. A beetle of mass m can crawl on the surface of the ball. Initially the ball is rotating with angular speed Ω with the beetle at the north pole. The beetle then walks (in any manner) to the equator of the ball and sits down. What is the angular speed of the ball now ?



3.A ball of mass M collides with a stick with moment of inertia $I = \beta m l^2$ (relative to its centre which is the CM). The ball is initially travelling with a speed V_0 perpendicular to the stick. The ball strikes the stick at a distance d from the centre. The collision is elastic. Find the resulting translational and rotational speeds of the stick, and also the resulting speed of the ball .

4. A homogeneous cube, each edge of which has a length l, is initially in a position of unstable equilibrium with one edge in contact with a horizontal plane. The cube is then given a small displacement and allowed to fall. Show that the angular velocity of the cube when one face strikes the plane is given by

$$\omega^2 = \frac{3}{2}\frac{g}{l}(\sqrt{2} - 1)$$

if the edge cannot slide on the plane.

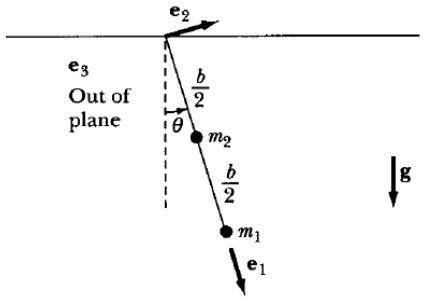
5. Show that none of the principal moments of inertia can exceed the sum of the other two.

6. If a physical pendulum (a rigid body) has the same period of oscillation when pivoted about either of two points of unequal distances from the centre of mass, show that the length of a simple pendulum with the same period is equal to the sum of the separations of the pivot points from the centre of mass. Such a pendulum is known as Kater's reversible pendulum.

7. What must be the ratio of the height to radius of a cylinder so that every axis is a principal axis

(with the CM as the origin)?

8. A cloud of interstellar gas of total mass M can move freely in space. Initially the cloud has the form of a uniform sphere of radius a rotating with an angular speed Ω about an axis through its centre. Later the cloud is observed to have changed its form to that of a thin circular disc of radius b which is rotating about an axis through its centre and perpendicular to its plane. Find the angular speed of the disc and the increase in kinetic energy.



9. Consider the pendulum shown in the figure composed of a rigid rod of length b with a mass m_1 at its end. Another mass m_2 is placed halfway down the rod. Find the frequency of small oscillations if the pendulum swings in a plane.