Physics I

Lecture 24

Looked at $\vec{P}$, $\vec{L}$ and E conservation for many particles

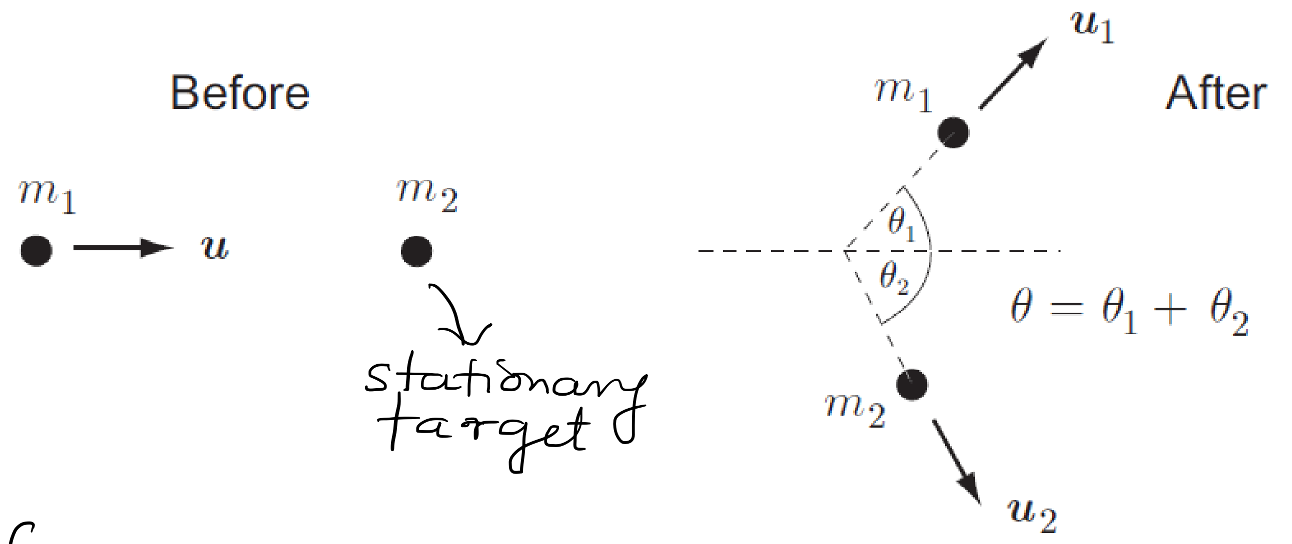
Today we will analyze collisions.

Collision processes

↳ Suppose that the mutual interactions between two particles $\longrightarrow 0$ as distance between them $\longrightarrow \infty$.

so far apart each moves with constant velocity

ex: collision of balls, Rutherford scattering



LAB frame , θ_1 = scattering angle

θ_2 = recoil angle .

Linear momentum is conserved

$$\boxed{m\vec{u} = m_1\vec{u}_1 + m_2\vec{u}_2} \quad \text{——} \quad (1)$$

$\Downarrow$ linear reln between $\vec{u}, \vec{u_1}, \vec{u_2}$ $\Rightarrow$ 3 velocities lie in a plane

2D problem

Collisions are not kinetic energy preserving in general

Cons. of energy

$$\boxed{\frac{1}{2} m_1 u^2 + Q = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2} \quad \text{--- (2)}$$

$Q \rightarrow$ energy gained (lost) in collision

Elastic collisions $\Rightarrow$ K.E conserved.

$\downarrow$ Is the notion of "elastic collisions" frame invariant?

Recall that we proved that $\left[v_i' = \vec{v}_i - \vec{V} \right]$

$\frac{1}{2} \sum_i m_i v_i'^2$.

$$T = \underbrace{T^{CM}}_{\frac{1}{2} M V^2} + \underbrace{T^{G}}$$

T^G → K.E of motion about the CM.

T^G → frame independent.

$\frac{1}{2} M V^2$ ⇓ preserved in collision.

(not affected by mutual interaction)

Hence the notion of "elastic collision" is indeed frame independent

Elastic Collisions

$$\frac{1}{2} m_1 u^2 = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 \quad — \quad 3$$

mom. cons.

$$m_1 \vec{u} = m_1 \vec{u_1} + m_2 \vec{u_2} \quad — \quad 1$$

Take scalar product of each side of 1

$$m_1^2 u^2 = m_1^2 u_1^2 + m_2^2 u_2^2 + 2 m_1 m_2 \vec{u_1} \cdot \vec{u_2} \quad — \quad 4$$

eliminate u^2 between 4 & 3

$$2 m_1 \vec{u_1} \cdot \vec{u_2} = (m_1 - m_2) u_2^2 \quad — \quad 5$$

$$2m_1 \vec{u_1} \cdot \vec{u_2} = (m_1 - m_2) u_2^2$$

$$2m_1 u_1 u_2 \cos\theta = (m_1 - m_2) u_2^2$$

$$\boxed{\cos\theta = \frac{(m_1 - m_2)}{2m_1 u_1} u_2} \quad \text{—} \quad (6)$$

provided $u_1 \neq 0$.

θ = opening angle $= \theta_1 + \theta_2$

Ex: Ball of mass m energy E in an elastic collision of mass 4m, initially at rest.

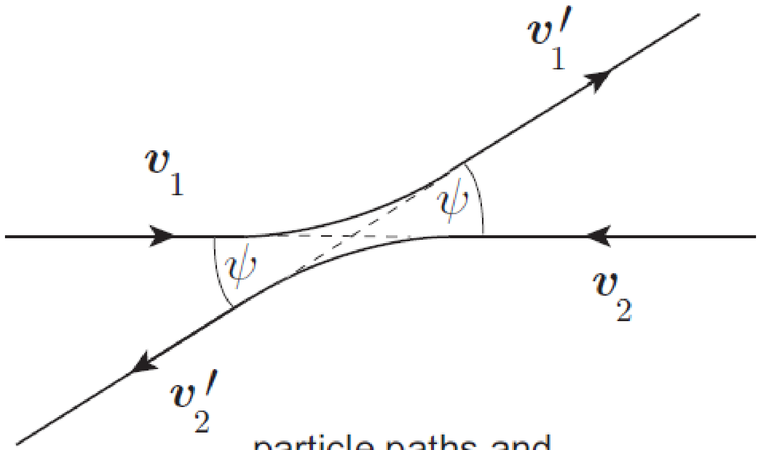
Observed; Two balls depart at 120° to each other.

$\frac{E_1}{E_2} = ?$ E_1, E_2 final energies

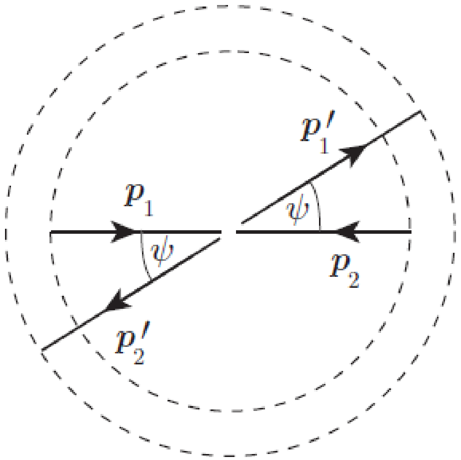
$$\cos\theta = \frac{(m_1 - m_2)\,u_2}{2 m_1 u_1} \implies \boxed{\frac{u_1}{u_2} = 3}$$

$$\frac{E_1}{E_2} = \frac{\frac{1}{2} m\, u_1^2}{\frac{1}{2}\, 4m\, u_2^2} = \frac{9}{4}$$

Collision process in CM/ZM frame $\longrightarrow$ CM is at rest.



particle paths and
initial and final velocities



initial and final momenta

Two particles, isolated system, CM moves with constant velocity. So the frame in which CM $\equiv$ G at rest is an inertial frame.

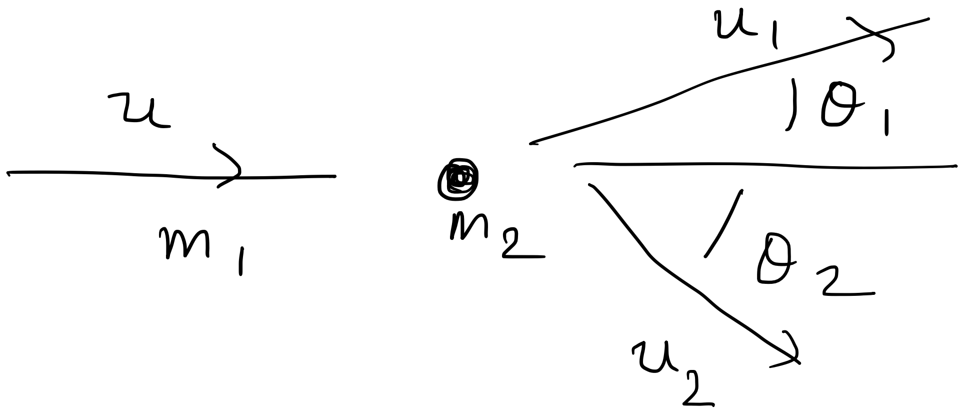
$$\vec{R} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}, \quad \vec{V} = \frac{m_1 \vec{V}_1 + m_2 \vec{V}_2}{m_1 + m_2} = 0$$

$$\Rightarrow \boxed{\vec{p}_1 + \vec{p}_2 = 0} \longrightarrow \text{Zero momentum frame.}$$

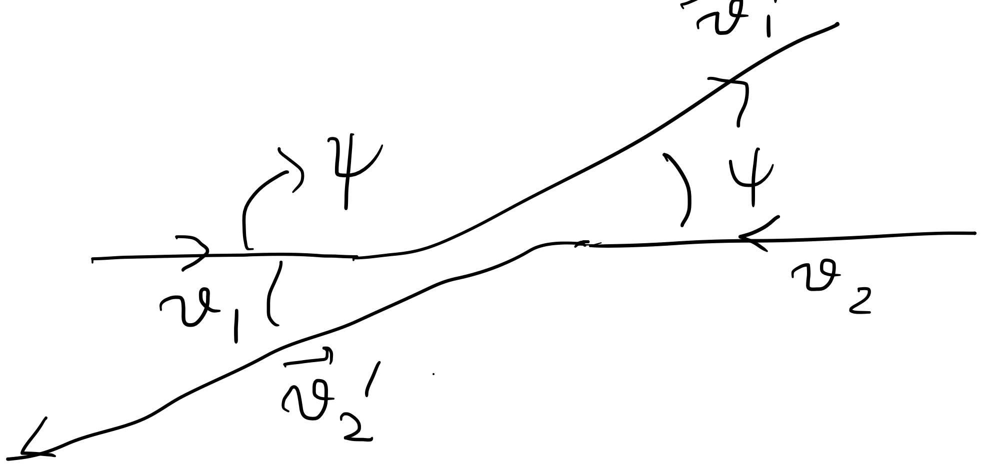
before $\vec{p_1} + \vec{p_2} = 0$

after $\vec{p_1}' + \vec{p_2}' = 0$

$\Rightarrow$ ZM frame.



LAB



ZM.

$\hookrightarrow$ each particle deflected through SAME angle ψ

total linear mom $\vec{P} = m_1 \vec{u}$.

$\vec{V} = \dfrac{m_1 \vec{u}}{m_1 + m_2}$

vel of CM