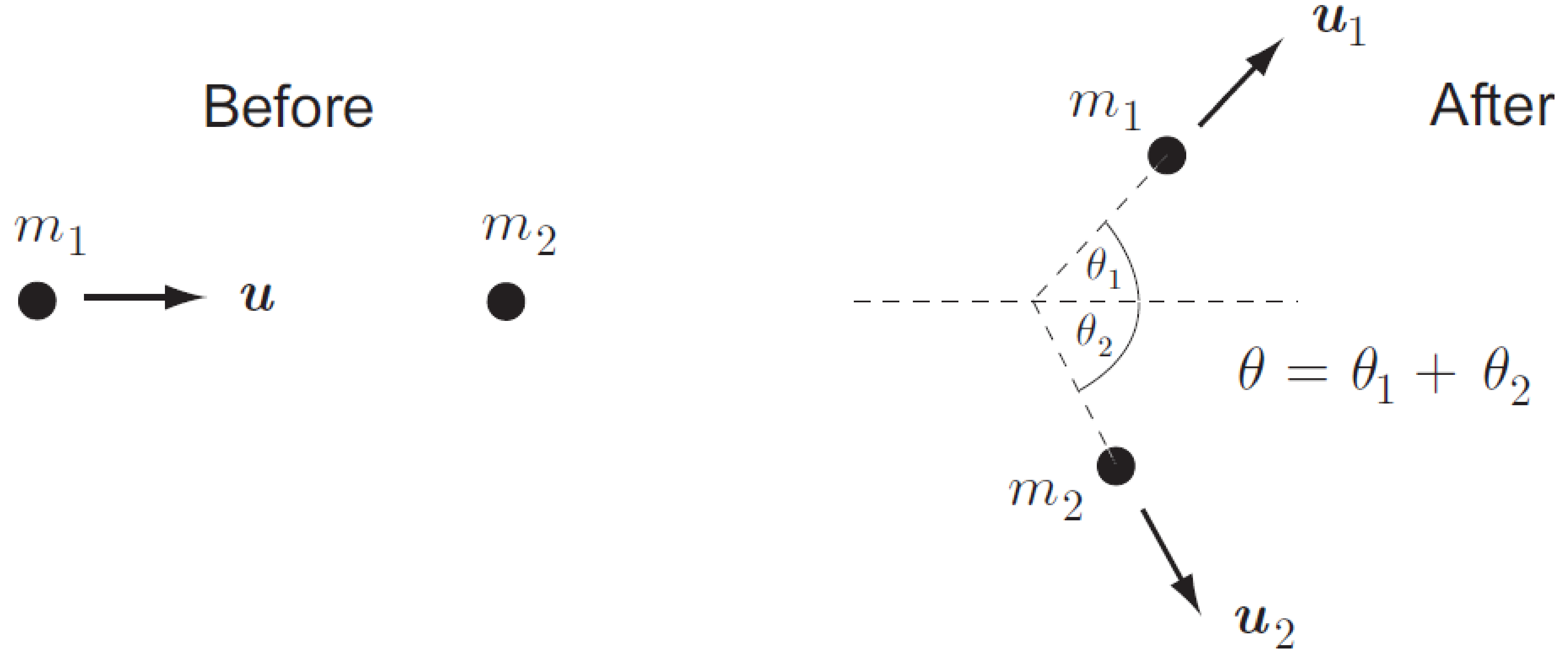
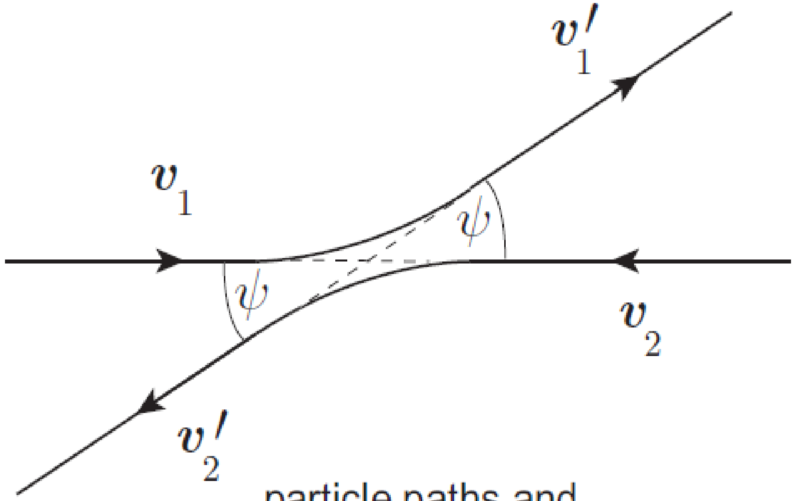
Physics I

Lecture 25

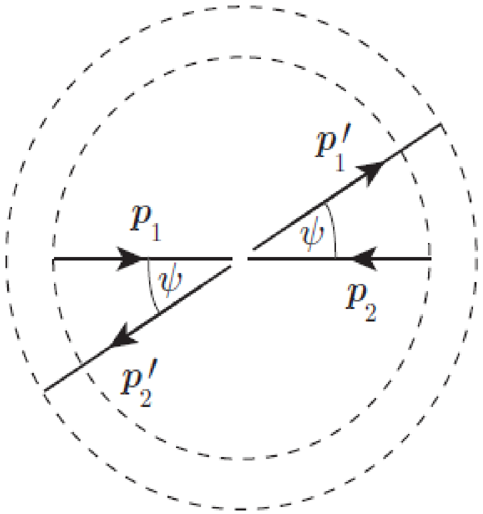
Lab frame


Before
m_1
$\boldsymbol{u}$
m_2
After
m_1
$\boldsymbol{u}_1$
θ_1
θ_2
$\theta = \theta_1 + \theta_2$
m_2
$\boldsymbol{u}_2$

ZM frame



particle paths and
initial and final velocities



initial and final momenta

$$\vec{V} = \frac{m_1 \vec{u}}{m_1 + m_2}$$

vel. of CM rel to lab frame.

Lab frame $\vec{P} = m_1 \vec{u} = (m_1 + m_2)\vec{V}$

$\vec{p}_1 = m_1 \vec{v}_1$, $\vec{p}_2 = m_2 \vec{v}_2$, $\vec{p}_1{}' = m_1 \vec{v}_1{}'$, $\vec{p}_2{}' = m_2 \vec{v}_2{}'$

$$\boxed{\vec{p}_1 + \vec{p}_2 = 0 \; ; \; \vec{p}_1{}' + \vec{p}_2{}' = 0}$$

each particle deflected through SAME angle ψ.

Conservation of energy

$$\frac{1}{2} m_1 |\vec{v}_1|^2 + \frac{1}{2} m_2 |\vec{v}_2|^2 + Q = \frac{1}{2} m_1 |\vec{v}_1'|^2 + \frac{1}{2} m_2 |\vec{v}_2'|^2$$

Let p be the magnitude of initial common momentum

Let p' " " " " final " "

Cons. of energy

$$\boxed{\frac{p^2}{2m_1} + \frac{p^2}{2m_2} + Q = \frac{p'^2}{2m_1} + \frac{p'^2}{2m_2}}$$

for elastic collisions $Q = 0$, $p = p'$

(p, ψ) determine final momenta $\vec{p}_1, \vec{p}_2$.

$$\frac{p^2}{2m_1} + \frac{p^2}{2m_2} + Q = \frac{p'^2}{2m_1} + \frac{p'^2}{2m_2}$$

$$\boxed{p'^2 = p^2 + \left(\frac{2Q m_1 m_2}{m_1 + m_2}\right)}$$

$$Q = 0 \Rightarrow p'^2 = p^2 \ , \ p' = p .$$

In a typical scattering problem what is known are masses m_1, m_2 and initial $\vec{p_1}, \vec{p_2}$.

$$\boxed{\begin{aligned} \vec{v_1} &= \vec{u} - \vec{V} \\ \vec{v_2} &= -\vec{V} \end{aligned}}$$

$\rightarrow$ connection between Lab & ZM initial velocities.

$$\vec{V} = \frac{m_1 \vec{u}}{m_1 + m_2}.$$

Initial momentum in ZM frame.

$$p = m_2 v_2$$

noting that $|\vec{v_2}| = |\vec{V}|$.

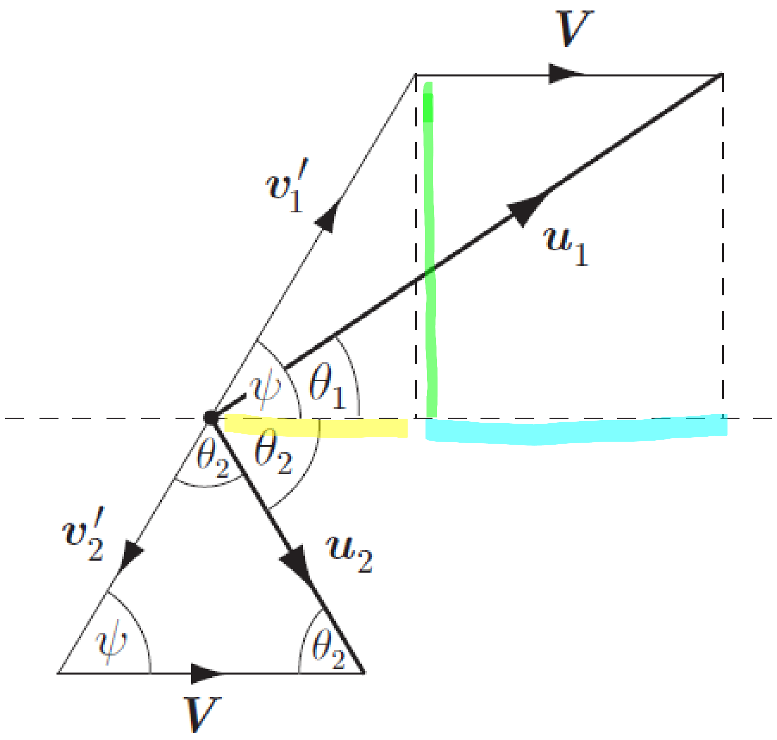
$$\boxed{p = \frac{m_1 m_2 u}{m_1 + m_2}}$$

$$p = m_2 \vec{v_2} = m_2 \vec{v_2}'.$$

Returning to Lab frame

elastic collisions. $Q = 0$.

$p' = p$.



$$\left.\begin{aligned} \vec{u}_1 &= \vec{v}_1' + \vec{V} \\ \vec{u}_2 &= \vec{v}_2' + \vec{V} \end{aligned}\right\}$$

$$v_1' = \frac{m_2 u}{m_1+m_2} \; ; \; v_2' = \frac{m_1 u}{m_1+m_2} = V. \quad (*)$$

$$\tan\theta_1 = \frac{v_1' \sin\psi}{v_1'\cos\psi + V} = \frac{\sin\psi}{\cos\psi + V/v_1'}$$

$$\boxed{\theta_2 = \frac{1}{2}(\pi - \psi)}$$

$$\boxed{\tan\theta_1 = \frac{\sin\psi}{\cos\psi + m_1/m_2}}$$

opening angle $\theta = \theta_1 + \theta_2$

$$\tan\theta = \tan(\theta_1+\theta_2) = \frac{\tan\theta_1 + \tan\theta_2}{1-\tan\theta_1\tan\theta_2} = \left(\frac{m_1+m_2}{m_1-m_2}\right)\cot\frac{\psi}{2}$$

↳ supplement intermediate steps

To find the final energies.

$$\vec{u}_2 = \vec{v}_2{}' + \vec{V}$$

$$u_2^2 = v_2^2 + V^2 + 2\vec{v}_2{}'\cdot\vec{V}$$

$$= 2V^2 - 2V^2\cos\psi$$

$$= 4V^2\sin^2\psi/2$$

$$\boxed{u_2 = 2V\sin\frac{\psi}{2}}$$

Final energies

$$\frac{E_2}{E_0} = \frac{\frac{1}{2} m_2 u_2^2}{\frac{1}{2} m_1 u^2} = \frac{\frac{1}{2} m_2 \left(2V \sin \psi/2\right)^2}{\frac{1}{2} m_1 u^2}.$$

$$\boxed{\frac{E_2}{E_0} = \frac{4\, m_1/m_2}{(m_1/m_2 + 1)^2} \sin^2 \frac{\psi}{2}}.$$

Recall $V = \dfrac{m_1 u}{m_1 + m_2}$

$\gamma = \frac{m_1}{m_2}$

1. $\tan\theta_1 = \frac{\sin\psi}{\cos\psi + \gamma}$

2. $\theta_2 = \frac{1}{2}(\pi - \psi)$.

3. $\tan\theta = \left(\frac{\gamma+1}{\gamma-1}\right)\cot\frac{\psi}{2}$

4. $\frac{E_2}{E_0} = \frac{4\gamma}{(\gamma+1)^2}\sin^2\psi/2$.