Physics I

Lecture 26

Quiz question

$\vec{F} = -k\vec{r}$. $\rightarrow U = \frac{1}{2}kr^2$

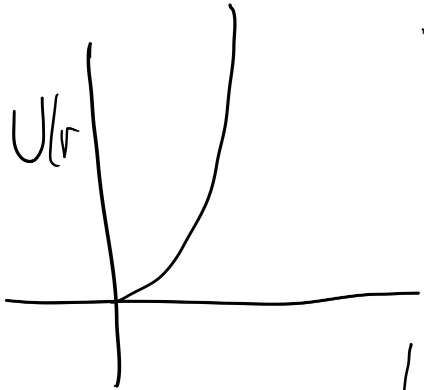
The particle can reach the origin.

$l = 0$

$mr^2\dot{\theta} = l = 0$

$\theta = \text{const}$.

meant to say can "always" reach the origin.



$$E = \frac{1}{2}m\dot{r}^2 + \frac{l^2}{2mr^2} + U(r)$$

$$\frac{1}{2}m\dot{r}^2 = E - \frac{l^2}{2mr^2} - U(r) > 0.$$

$$\left(Er^2\right)_{r\to 0} - \frac{l^2}{2m} - \left(U(r)r^2\right)_{r\to 0} > 0.$$

$$\left(U(r)r^2\right)_{r\to 0} < -\frac{l^2}{2m}.$$

e.g

$U(r) = -\frac{\alpha}{r^n}$

$n > 2$. $n = 2$, $\alpha < l^2/2m$ will reach centre

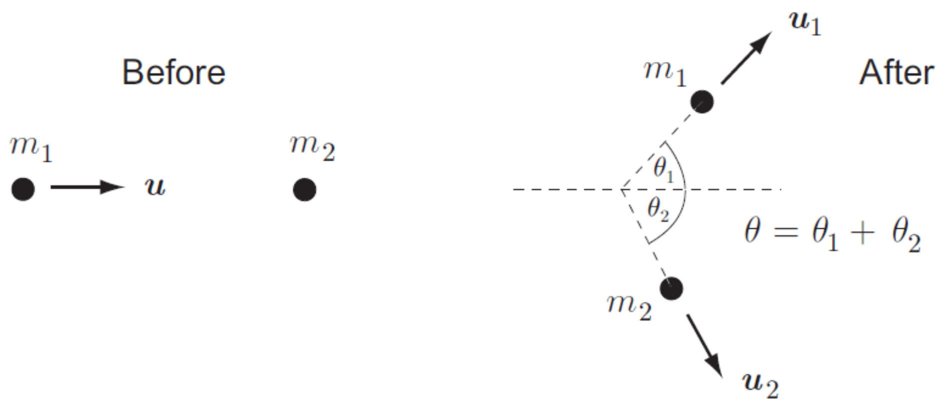
Elastic collision formulae

$$\text{A.}\quad \tan\theta_1 = \frac{\sin\psi}{\cos\psi + \gamma} \qquad\qquad \text{B.}\quad \theta_2 = \tfrac{1}{2}(\pi - \psi)$$

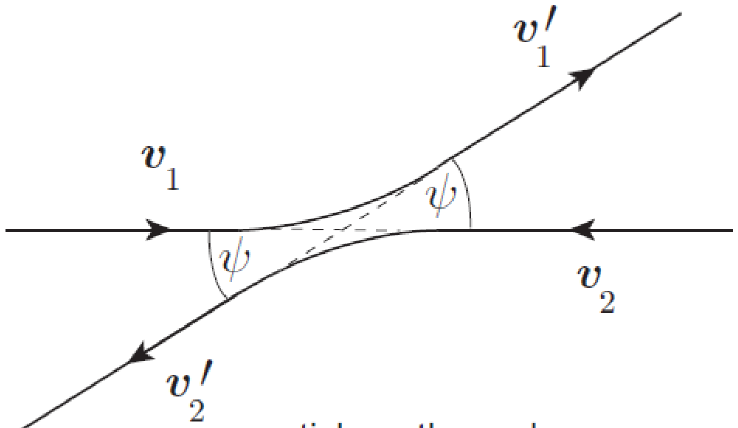
$$\text{C.}\quad \tan\theta = \left(\frac{\gamma+1}{\gamma-1}\right)\cot(\tfrac{1}{2}\psi) \qquad\qquad \text{D.}\quad \frac{E_2}{E_0} = \frac{4\gamma}{(\gamma+1)^2}\sin^2(\tfrac{1}{2}\psi) \tag{10.22}$$

ψ is the scattering angle in the ZM frame, and $\gamma = m_1/m_2$, the mass ratio of the two particles.

LAB



ZM .



particle paths and
initial and final velocities

In an experiment, particles of mass m and energy E are used to bombard stationary target particles of mass 2m. The experimenters wish to select particles that after scattering have an energy E/3. At which scattering angle will they find such particles ?

$E_2/E_0 = 2/3$.

$\theta_1 = ?$ $\frac{E_1}{E_0} = \frac{1}{3}$, $\frac{E_2}{E_0} = \frac{4\gamma}{(\gamma+1)^2} \sin^2 \psi/2$.

$\gamma = \frac{1}{2}$.

$\tan\theta_1 = \frac{\sin\psi}{\cos\psi + \gamma}$.

$\frac{2}{3} = \frac{4 \times \frac{1}{2}}{9} \times 4 \sin^2 \psi/2$

$\Rightarrow \infty$

$\sin^2 \psi/2 = \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{9}}{\cancel{8}} \frac{3}{4}$; $\sin\frac{\psi}{2} = \frac{\sqrt{3}}{2}$.

$\boxed{\theta_1 = 90^\circ}$

$\boxed{\psi = 120^\circ}$.

In one collision, the opening angle was 45 degrees. What are the individual scattering angles ?

$$r = \frac{1}{2} \quad , \quad \theta_1 = ? \quad , \quad \theta_2 = ?$$

$$\tan\theta = \left(\frac{r+1}{r-1}\right) \cot \psi/2$$

$$1 = \frac{\frac{1}{2}+1}{\frac{1}{2}-1} \cot \psi/2 \Rightarrow \boxed{\cot\frac{\psi}{2} = -\frac{1}{3}}$$

$$\psi/2 = 72^\circ$$

$$\theta_2 = \frac{1}{2}(\pi - \psi) = 90^\circ - 72^\circ = 18^\circ$$

$$\theta_1 = \theta - \theta_2 = 45^\circ - 18^\circ = 27^\circ$$

In another collision, the scattering angle was measured to be 45 degrees. What was the recoil angle ?

$$\gamma = \frac{1}{2}. \qquad \theta_1 = 45^\circ, \quad \theta_2 = ?$$

$$\tan\theta_1 = \frac{\sin\psi}{\cos\psi + \gamma} = 1.$$

I made an error in the computation of θ_2 in class which, was corrected later by Pratichi Paromita. I am uploading the corrected version

$$\frac{\sin\psi}{\cos\psi + \frac{1}{2}} = 1 \Rightarrow 2\sin\psi - 2\cos\psi = 1$$

$$\sqrt{8}\sin(\psi - 45^\circ) = 1$$

$$\psi \simeq 66^\circ, \Rightarrow \theta_2 = \frac{1}{2}(\pi - \psi) \simeq 56^\circ$$

In an elastic collision between an alpha particle and an unknown nucleus at rest the alpha particle was deflected through a right angle and lost 40% of its energy. Identify the mystery nucleus.

let the mass of unknown nucleus = M

$\gamma = \frac{4}{M}$

$M = 16$

$\rightarrow$ Oxygen

Elastic collision formulae

A. $\tan\theta_1 = \dfrac{\sin\psi}{\cos\psi + \gamma}$

B. $\theta_2 = \frac{1}{2}(\pi - \psi)$

C. $\tan\theta = \left(\dfrac{\gamma+1}{\gamma-1}\right)\cot(\frac{1}{2}\psi)$

D. $\dfrac{E_2}{E_0} = \dfrac{4\gamma}{(\gamma+1)^2}\sin^2(\frac{1}{2}\psi)$

(10.22)

ψ is the scattering angle in the ZM frame, and $\gamma = m_1/m_2$, the mass ratio of the two particles.

$\frac{E_1}{E_0}$

Motion in a Noninertial Reference Frame

- Newton's Laws are valid ***only*** in ***inertial*** frames
- However, there are problems where treating motion of the system in a non-inertial frames is simpler
- For example, to describe the motion of a body on earth, or near earth, it might be useful to use a coordinate system fixed on earth. This is clearly a non-inertial frame, since the earth rotates.
- To describe the motion of a rigid body which is free to rotate and accelerate, it is often convenient to use a reference frame fixed to the rigid body.