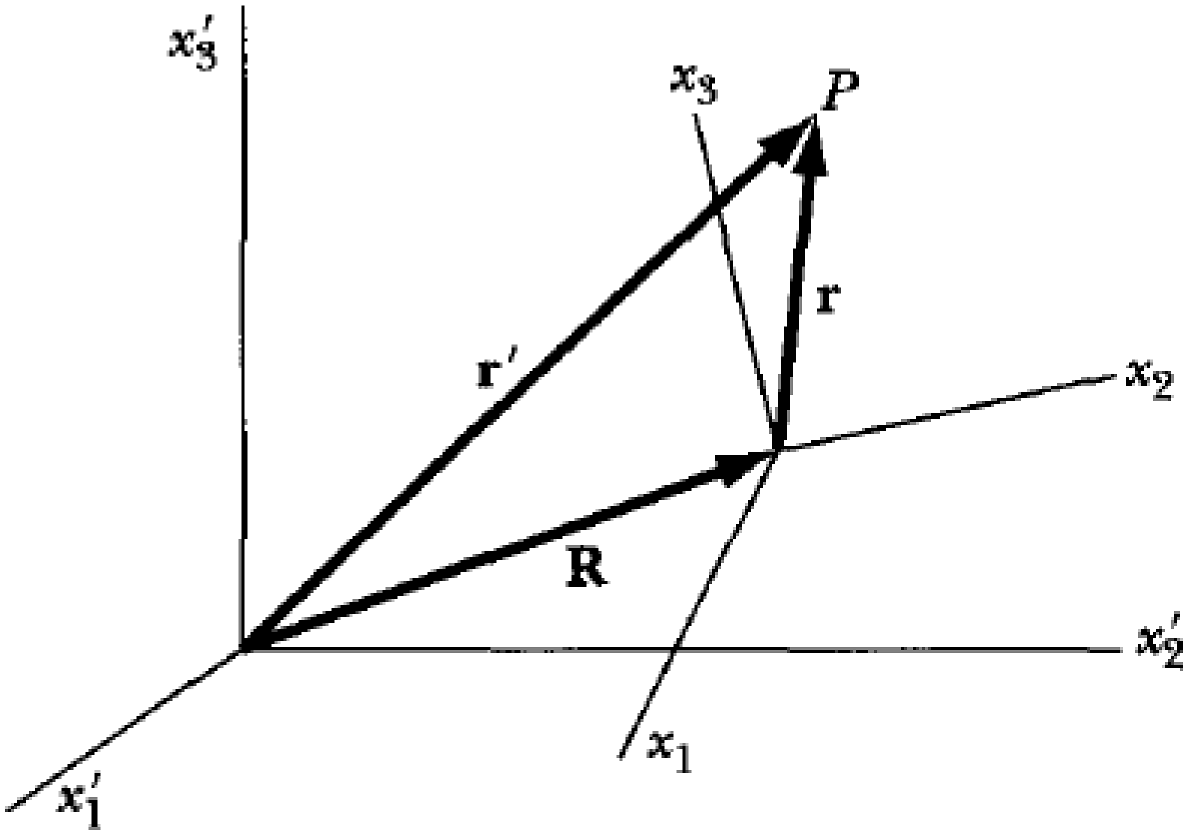
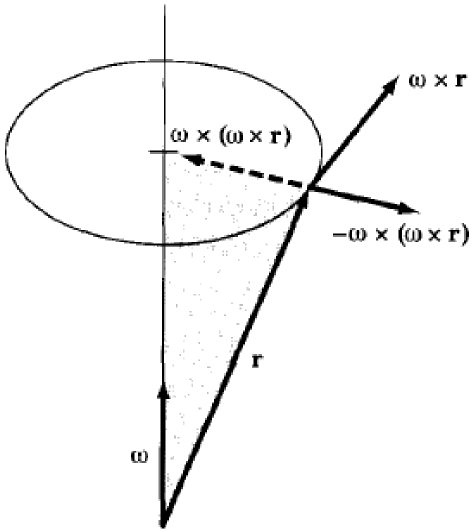
Physics I

Lecture 28



x_3'
x_3
P
$\mathbf{r}'$
$\mathbf{r}$
x_2
$\mathbf{R}$
x_2'
x_1
x_1'

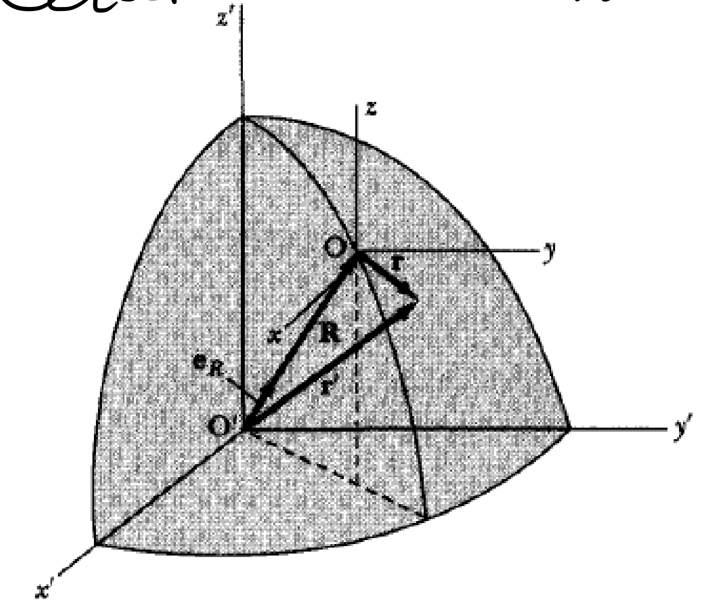


$$\vec{F}_{eff} = \vec{F} - m\ddot{\vec{R}}_f - m\dot{\vec{\omega}}\times\vec{r} - m\vec{\omega}\times(\vec{\omega}\times\vec{r}) - 2m\vec{\omega}\times\vec{v}_r \quad \text{--- } (1)$$

$$\vec{F}_{eff} = m\vec{a}_r \; ; \; \vec{F} = m\vec{a}_f \quad \text{--- } (2)$$

$$\vec{F}_{eff} = m\vec{a}_f + (\text{non-inertial terms}) \quad \text{--- } (3)$$

Motion Relative to Earth



In order to study the motion of an object near Earth's surface, we place a fixed inertial frame $x'y'z'$ at the center of Earth and the moving frame xyz on Earth's surface.

$\vec{F} \longrightarrow$ forces measured w.r.t fixed inertial frame

$$= \vec{S} + m\vec{g}_0$$

$\vec{S} \hookrightarrow$ external forces other than gravitational

$$\vec{g}_0 = -\frac{GM_E}{R^2}\hat{e}_R \quad \text{——} \quad (5)$$

$$\vec{F}_{eff} = \vec{S} + m\vec{g}_0 - m\ddot{\vec{R}}_f - m\underbrace{\dot{\vec{\omega}}\times\vec{r}} - m\vec{\omega}\times(\vec{\omega}\times\vec{r})$$
$$-2m\vec{\omega}\times\vec{v}_r \quad \text{—— (6)}$$

neglected ($\because\ \dot{\vec{\omega}} = 0$).

$\vec{\omega}$ is in z' direction ; $\omega = 7.3\times10^{-5}$ rad/s.

$\vec{\omega}$ is practically constant, $\dot{\vec{\omega}} = 0$.

Recall

$$\left(\frac{d\vec{Q}}{dt}\right)_{fixed} = \left(\frac{d\vec{Q}}{dt}\right)_{rotating} + \vec{\omega}\times\vec{Q} \quad \text{—— (7)}.$$

$$\therefore\ \ddot{\vec{R}}_f = \vec{\omega}\times\dot{\vec{R}}_f \quad \text{—— (8)}$$

$$\vec{F}_{eff} = \vec{S} + m\vec{g}_0 - m\vec{\omega}\times\left[\vec{\omega}\times(\vec{r}+\vec{R})\right] - 2m\vec{\omega}\times\vec{v}_r \quad \text{—— (9)}$$

$$\vec{F}_{eff} = \vec{S} + m\underbrace{\vec{g}_0 - m\,\vec{\omega}\times\left[\vec{\omega}\times(\vec{r}+\vec{R})\right]} - 2m\,\vec{\omega}\times\vec{v}_r \quad \text{--- (9)}$$

effective $\vec{g}$ measured on earth.

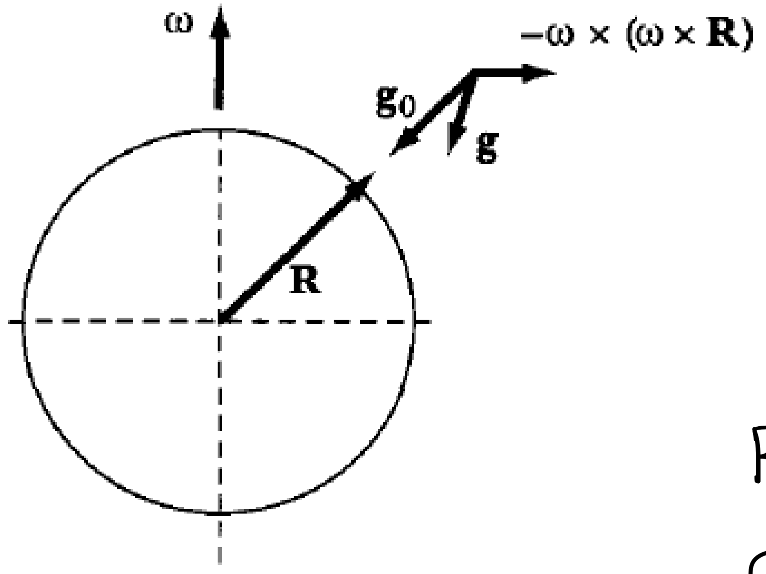
$$\vec{g} = \vec{g}_0 - \vec{\omega}\times\left[\vec{\omega}\times(\vec{r}+\vec{R})\right] \quad \text{--- (10)}$$

$r \ll R \qquad \simeq \vec{\omega}\times(\vec{\omega}\times\vec{R})$

$$\boxed{\vec{F}_{eff} = \vec{S} + m\vec{g} - 2m\,\vec{\omega}\times\vec{v}_r} \quad \text{--- (11)}$$

Period of pendulum will determine mag. of g.

direction $\longrightarrow$ direction of a plumb bob.



$\omega^2 R = 0.034\ m/s^2$

0.35% of g.

Relative magnitudes of centrifugal vs coriolis.

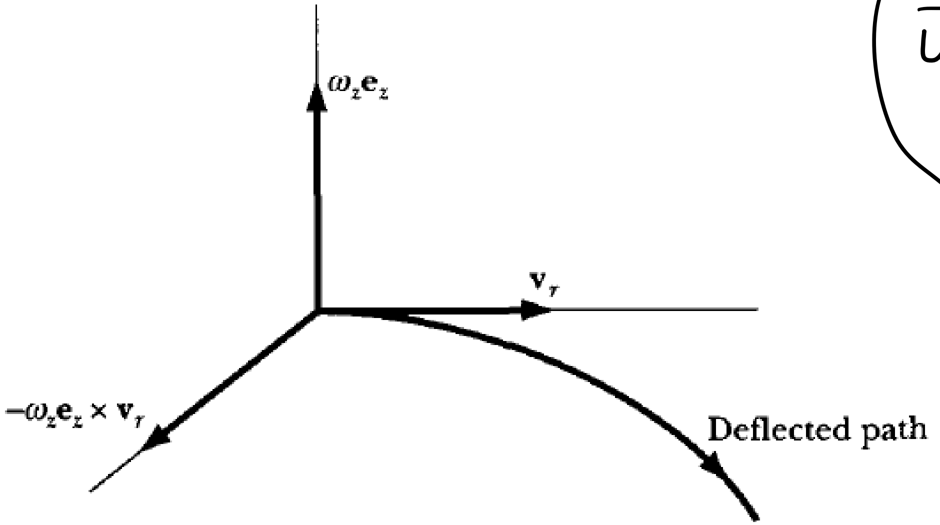
$F_{cf} \sim m R \omega^2$.

$F_c \sim m v \omega$.

$$\frac{F_{cor.}}{F_c} \sim \frac{v}{R\omega} \sim \frac{v}{V} \sim \frac{v}{500\ m/s}.$$

$v > 1800\ km/hr$ Coriolis force is imp.

Coriolis force effect



$\vec{\omega}$: directed in northerly direction

Northern hemisphere.
$\vec{\omega}$ has a component ω_z directed outward along local vertical.

If a particle is projected in a horizontal plane (in the local coord. system on surface of earth) $\vec{v}_r$

Coriolis force $= -2m\,\vec{\omega} \times \vec{v}_r$, has a component $2m\,\omega_z v_r$ directed towards the right

$\longrightarrow$ deflection to the right results.

Coriolis force depends on z-component of ω.

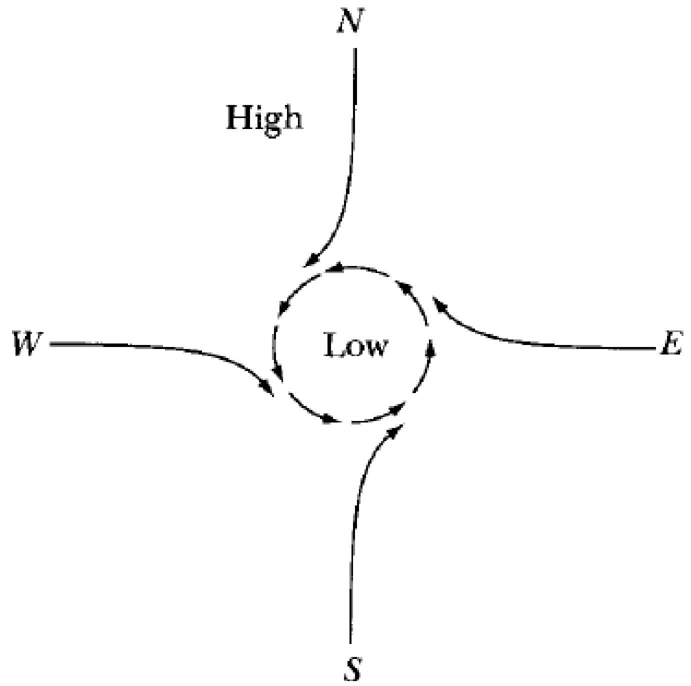
$\Rightarrow$ depends on latitude, maximum at N-pole

zero at equator.

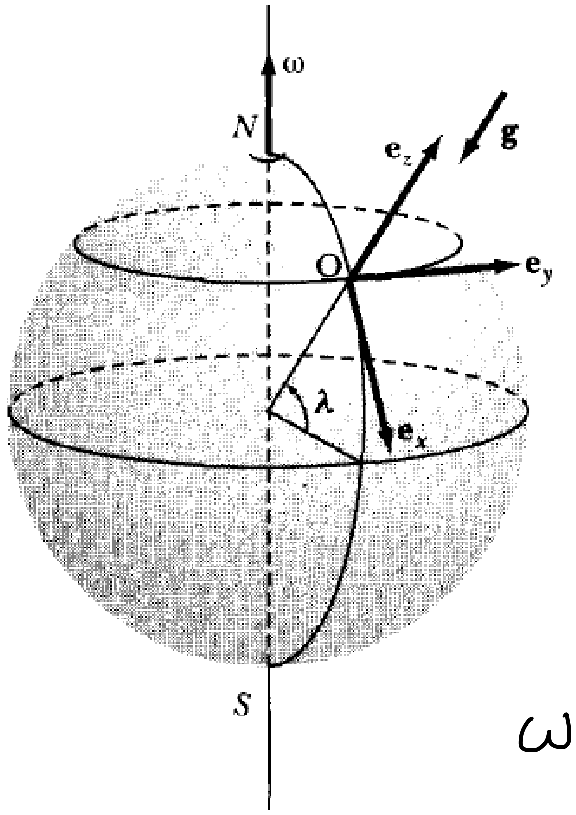
In the Southern Hemisphere, the component of ω_z is directed inwards. along the local vertical

$\Rightarrow$ all deflections will be to the left.

opposite to what happens in the N-hemisphere.



3 The Coriolis force deflects air in the Northern Hemisphere to the right producing cyclonic motion.



Horizontal deflection from the plumb line by the Coriolis force acting on a particle falling freely under earth's gravity.

$$\omega_x = -\omega\cos\lambda$$

$$\omega_y = 0$$

$$\omega_z = \omega\sin\lambda .$$

Eqn of motion

$$\ddot{x} = \underset{\simeq 0}{g_x} - 2(\underline{\vec{\omega} \times \vec{v}_r})_x$$

$$\ddot{y} = \underset{=0}{g_y} - 2(\vec{\omega} \times \vec{v}_r)_y$$

$$\ddot{z} = g_z - 2(\vec{\omega} \times \vec{v}_r)_z$$

C-force produces small vel components in $\hat{e}_x$, $\hat{e}_y$ directions

→ ignore them

The zeroth approx we make

$$\left.\begin{aligned} \dot{x} &\simeq 0 \\ \dot{y} &\simeq 0 \\ \dot{z} &\simeq -gt \end{aligned}\right\}$$

$$\vec{\omega} \times \vec{v}_r \simeq \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ -\omega\cos\lambda & 0 & -\omega\sin\lambda \\ 0 & 0 & -gt \end{vmatrix}$$

$$\boxed{\vec{\omega} \times \vec{v}_r \approx -(\omega g t \cos\lambda)\hat{e}_y}$$

$$g_x = 0$$

$$g_y = 0$$

$$g_z = -g.$$

eqn. of motion

a

$$\left.\begin{aligned} (a_r)_x &= \ddot{x} \simeq 0 \\ (a_r)_y &= \ddot{y} \simeq 2\omega g t \cos\lambda \\ (a_r)_z &= \ddot{z} \simeq -g. \end{aligned}\right\}.$$

Integrate

$$\boxed{y(t) \simeq \frac{1}{3}\omega g t^3 \cos\lambda}$$

$$[\, y = 0,\ \dot{y} = 0 \text{ at } t = 0 \,]$$

$$z(t) \simeq \underbrace{z(0)}_{h} - \frac{1}{2} g t^2.$$

time of fall $\quad t \simeq \sqrt{\dfrac{2h}{g}}.$

Eastward deflection

$$d \cong \frac{1}{3}\omega\, t^3 \cos\lambda \Big|_{t \cong \sqrt{\frac{2h}{g}}}$$

$$\boxed{d \cong \frac{1}{3}\omega \cos\lambda \sqrt{\frac{8h^3}{g}}}$$

$h \simeq 100\,m$ at latitude 45°.

$$\boxed{d \simeq 1.55\ cm}$$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