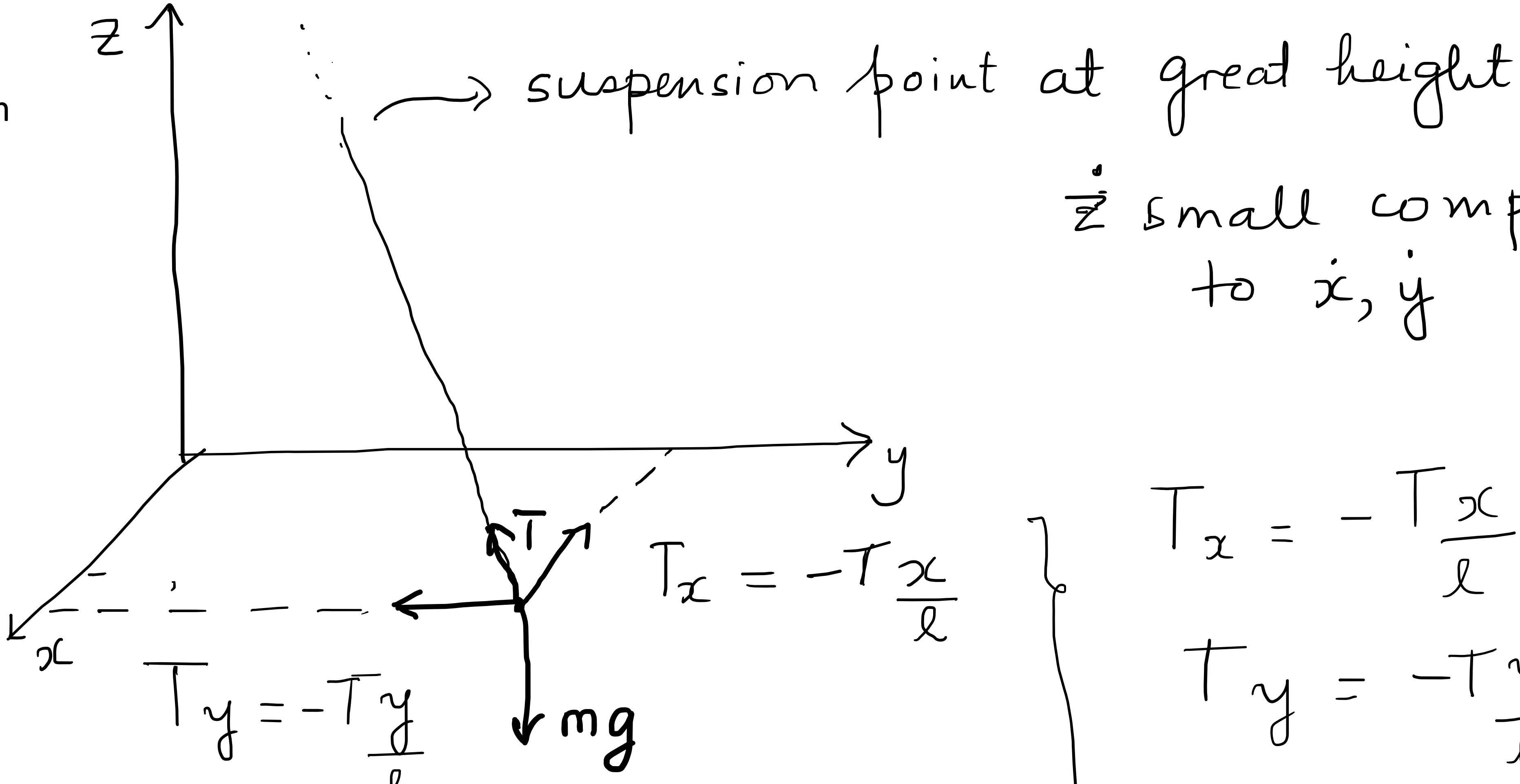
Physics I

Lecture 29

Foucault Pendulum



$$T_x = -T\frac{x}{l}$$

$$T_y = -T\frac{y}{l}$$

$$\vec{a}_r = \vec{g} + \frac{\vec{T}}{m} - 2\,\vec{\omega}\times\vec{v}_r .$$

$\dot{z}$ small compared to $\dot{x}, \dot{y}$

$$T_x = -T\frac{x}{l}$$

$$T_y = -T\frac{y}{l}$$

$$T_z \simeq T .$$

$\vec{g}$, $g_x = 0$, $g_y = 0$, $g_z = -g$

$\omega_x = -\omega\cos\lambda \longrightarrow \lambda$: latitude .

$\omega_y = 0$.

$\omega_z = \omega\sin\lambda$.

$(\vec{v}_r)_x = \dot{x}$

$(\vec{v}_r)_y = \dot{y}$

$(\vec{v}_r)_z \simeq \dot{z} \cong 0$

$$\vec{a}_r = \vec{g} + \frac{\vec{T}}{m} - 2\vec{\omega} \times \vec{v}_r \quad \text{———} \; (\ast)$$

$$\vec{\omega} \times \vec{v} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ -\omega\cos\lambda & 0 & \omega\sin\lambda \\ \dot{x} & \dot{y} & 0 \end{vmatrix}$$

$$\left.\begin{aligned} (\vec{\omega} \times \vec{v}_r)_x &\cong -\dot{y}\,\omega\sin\lambda \\ (\vec{\omega} \times \vec{v}_r)_y &\cong \dot{x}\,\omega\sin\lambda \\ (\vec{\omega} \times \vec{v}_r)_z &\cong -\dot{y}\,\omega\cos\lambda \end{aligned}\right\}$$

$$\vec{a}_r = \vec{g} + \frac{\vec{T}}{m} - 2\vec{\omega} \times \vec{v}_r .$$

$$\left.\begin{aligned} (\vec{a}_r)_x &= \ddot{x} \cong -\frac{T}{m}\frac{x}{l} + 2\dot{y}\omega\sin\lambda \\ (\vec{a}_r)_y &= \ddot{y} \cong -\frac{T}{m}\frac{y}{l} - 2\dot{x}\omega\sin\lambda \end{aligned}\right\} \begin{aligned} &— (1) \\ &— (2) \end{aligned}$$

for small displacements $T \cong mg$.

$\alpha^2 = \frac{T}{ml} \cong \frac{g}{l}$, $\omega_z = \omega\sin\lambda$; (1) & (2) become

$$\left.\begin{aligned} \ddot{x} + \alpha^2 x &\cong 2\omega_z \dot{y} \quad — (3) \\ \ddot{y} + \alpha^2 y &\cong -2\omega_z \dot{x} \quad — (4) \end{aligned}\right\}$$

multiply 2nd eqn. by i and add to 1st.

Becomes.

$$(\ddot{x}+i\ddot{y})+\alpha^2(x+iy) \simeq -2\omega_z(i\dot{x}-\dot{y})$$
$$\simeq -2i\omega_z(\dot{x}+i\dot{y}) \quad \text{---} \; (**)$$

$$q \simeq x+iy$$

(**) becomes.

$$\ddot{q}+2i\omega_z\dot{q}+\alpha^2 q = 0 \quad \text{---} \; (3)$$

$\hookrightarrow$ damped H.O. with pure imaginary damping coeff.

$$q(t) \cong e^{-i\omega_z t}\left[A e^{\sqrt{-\omega_z^2-\alpha^2}\,t} + B e^{-\sqrt{-\omega_z^2-\alpha^2}\,t}\right]$$

If the earth were not rotating $\omega_z = 0$.

$$\ddot{q}' + \alpha^2 q' \cong 0 \,,$$

$\alpha \Rightarrow$ oscillation freq. of pendulum $\gg \omega_z$.

$$q'(t) = x'(t) + i y'(t) = A e^{i\alpha t} + B e^{-i\alpha t}$$

$$q(t) = q'(t)\, e^{-i\omega_z t}.$$

$$x(t) + i y(t) = \left[x'(t) + i y'(t)\right] e^{-i \omega_z t}.$$

$$= \left[x'(t) + i y'(t)\right] \left(\cos\omega_z t - i \sin\omega_z t\right).$$

$$= \left(x' \cos\omega_z t + y' \sin\omega_z t\right)$$

$$+ \; i\left(-x' \sin\omega_z t + y' \cos\omega_z t\right)$$

$$\boxed{\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} \cos\omega_z t & \sin\omega_z t \\ -\sin\omega_z t & \cos\omega_z t \end{pmatrix} \begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix}}$$

$$\theta = \omega_z t = \omega \sin\lambda \, t.$$

Rigid Bodies

Example of many particle system is a rigid body.

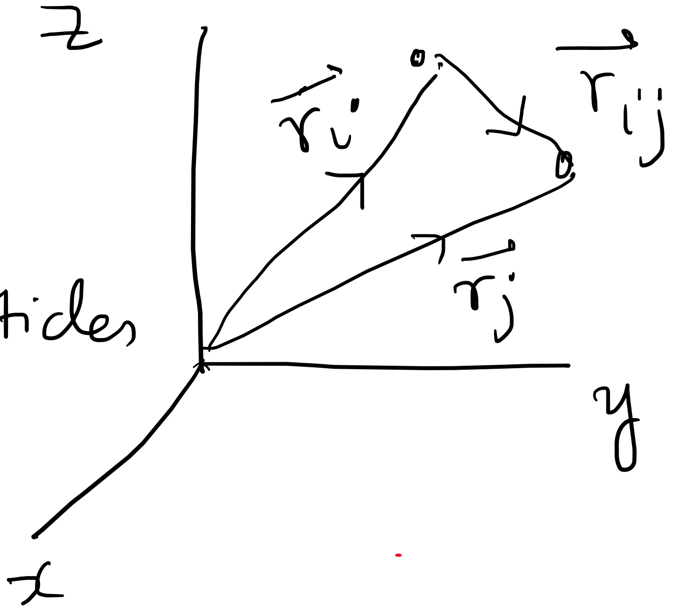
collection of particles whose relative distances are constrained to be fixed

Rigid body is an idealization

1. component particles undergo vibrations.
2. In special relativity relative distances are observer dependent.

Number of degrees of freedom of a rigid body

N particles



If all particles were allowed to move freely

$\#$ of degrees of freedom $= 3N$

rigid body constraints

$|\vec{r}_{ij}| = c_{ij} = \text{constant}$ — (1)

$\#$ of constraints from (1).

$= \frac{N(N-1)}{2}$.

$\#$ of free degrees of freedom $= 3N - \frac{N(N-1)}{2}$?

for large N < 0

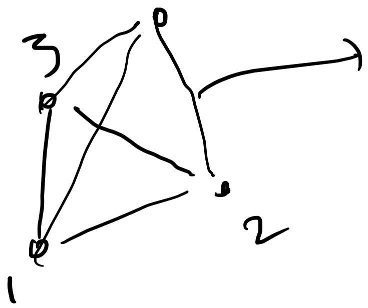
The constraints

$|\vec{r}_{ij}| = c_{ij}$ are not all independent

So what are the true # of degrees of freedom

$\boxed{2, 5 \; \ldots ??}$

One needs to fix coordinates of only 3 non colinear particles



$9 - 3 = 6$