Physics I

Lecture 3

Third Law

$$\vec{F}_{12} = -\vec{F}_{21}$$

strong form

Force acts along line joining two particles.

central forces.

$$\boxed{\vec{p}_1 + \vec{p}_2 = \vec{P}}$$

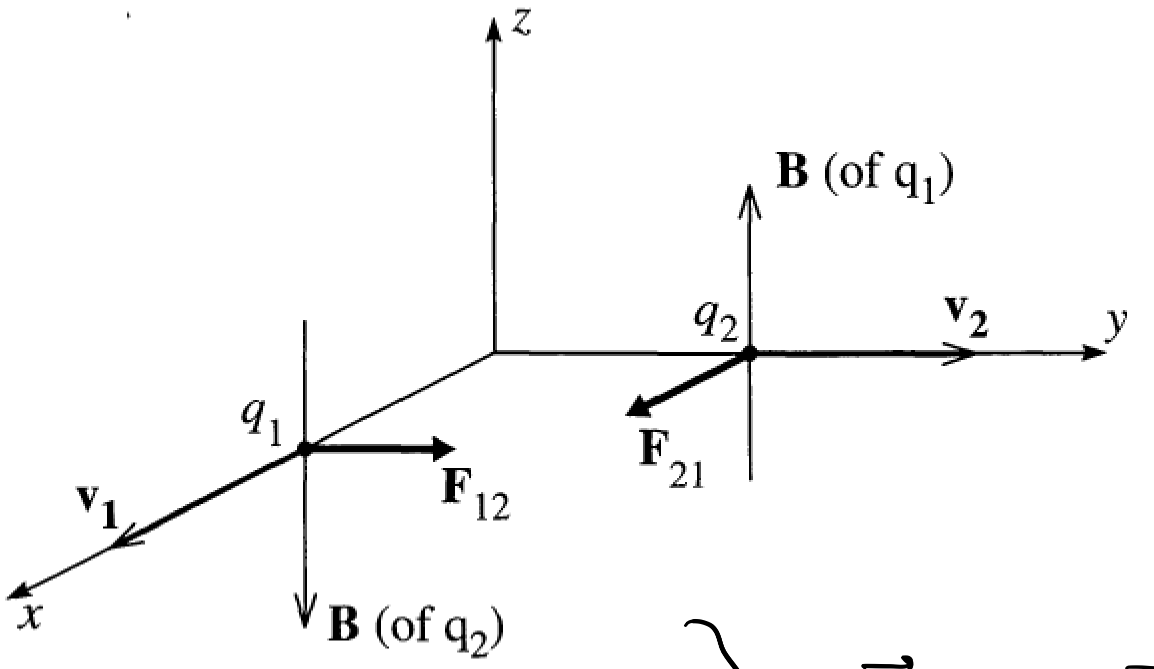
$$\boxed{\vec{P} = \text{const} \quad \text{when } \vec{F}_{ext} = 0}$$

Conservation of linear momentum

$\vec{F}_{12}(t) = -\vec{F}_{21}(t)$

Third Law is not always valid

↓ moving charges mag fields



$\vec{F}_{mag} = q(\vec{v} \times \vec{B})$

$\vec{F}_{12} \neq -\vec{F}_{21}$

Coulomb force $q\vec{E}$ is central
acts along line joining charges
obeys third law.

↳ Mom conservation is violated!!

Electromag fields carry momentum

Turns ~~or~~ out that

Particle + field momentum is indeed conserved.

↓ full theory of electromagnetism

$v \ll c$ violation is negligible.

mag field contribution is $\frac{v^2}{c^2}$ (Coulomb force)

Basic Problem

$$\vec{F} = m\ddot{\vec{r}}$$

3   2^{nd} order diff eqn.s, in principle coupled.

integrate to find

$\vec{r} = \vec{r}(t)$, when $\vec{F}(\vec{r}, \vec{v}, t)$ is known

given initial conditions

$\vec{r}(0) = \vec{r}_0$

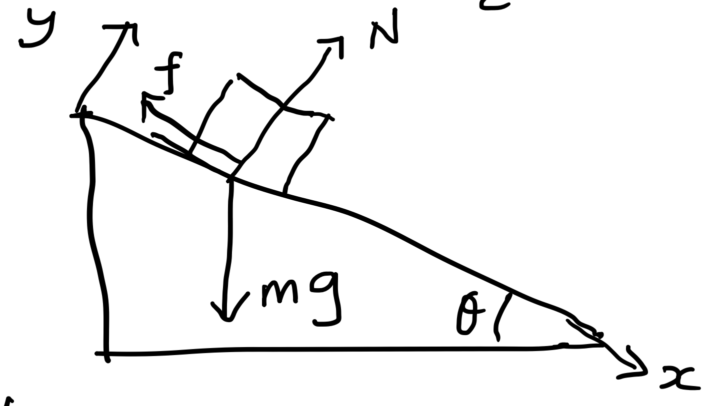
$\vec{v}(0) = \vec{v}_0$

In cartesian coordinates

$$m\ddot{x} = F_x$$

$$m\ddot{y} = F_y$$

$$m\ddot{z} = F_z$$

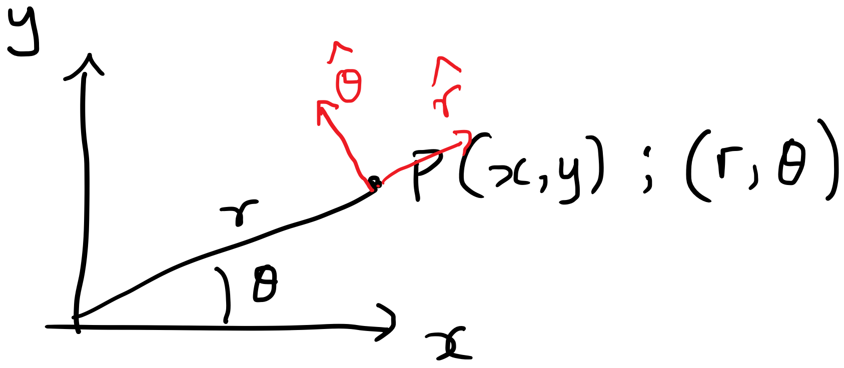


$x(t) = ?$

$y(t) = ?$

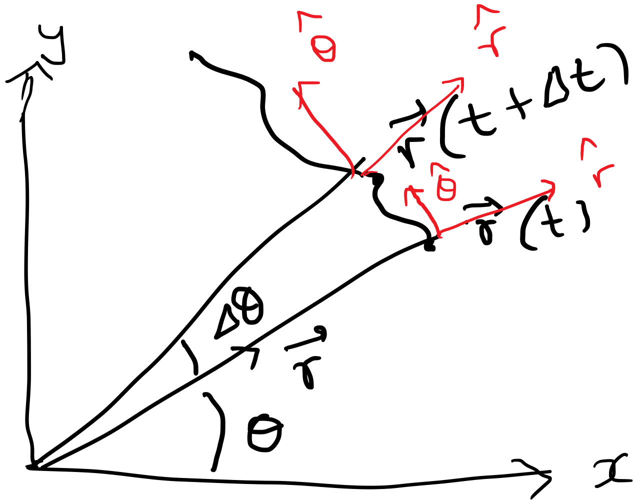
↳ sample problem

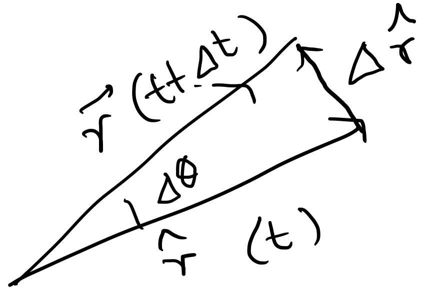
2D polar coordinates



$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$
$$r = \sqrt{x^2+y^2} \,.$$

$$\vec{r} = x\,\hat{x} + y\,\hat{y}$$
$$\vec{r} = r\,\hat{r}$$
$$\vec{F} = F_r\,\hat{r} + F_\theta\,\hat{\theta} \,.$$





$$\Delta\hat r \simeq \Delta\theta\,\hat\theta$$

$$\lim_{\Delta t\to 0}\frac{\Delta\hat r}{\Delta t} \simeq \frac{\Delta\theta}{\Delta\theta}\,\hat\theta$$

$$\boxed{\dot{\hat r} = \dot\theta\,\hat\theta}$$

$$\dot{\vec r} = \frac{d}{dt}(r\,\hat r)$$

$$\vec v = \frac{dr}{dt}\hat r + r\frac{d\hat r}{dt}$$

$$= \underbrace{\dot r}\,\hat r + r\dot\theta\,\hat\theta$$

$$\boxed{\begin{aligned}\vec v &= v_r\,\hat r + v_\theta\,\hat\theta\\ &= \dot r\,\hat r + r\dot\theta\,\hat\theta\end{aligned}}$$

$$v_r = \dot r$$

$$v_\theta = r\dot\theta$$

Alternatively,

$$\hat{r} = \cos\theta\,\hat{x} + \sin\theta\,\hat{y}$$

$$\hat{\theta} = -\sin\theta\,\hat{x} + \cos\theta\,\hat{y}$$

$$\dot{\hat{r}} = -\sin\theta\,\dot{\theta}\,\hat{x} + \cos\theta\,\dot{\theta}\,\hat{y}$$

$$= \dot{\theta}(-\sin\theta\,\hat{x} + \cos\theta\,\hat{y})$$

$$\boxed{\dot{\hat{r}} = \dot{\theta}\,\hat{\theta}} \qquad \boxed{\dot{\hat{\theta}} = -\dot{\theta}\,\hat{r}}$$

$$\ddot{\vec{r}} = \frac{d}{dt}(\dot{r}\hat{r} + r\dot{\theta}\hat{\theta}).$$

$$= \ddot{r}\,\hat{r} + \dot{r}\dot{\theta}\,\hat{\theta} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} - r\dot{\theta}^2\hat{r}$$

$$\boxed{\ddot{\vec{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{\theta}}$$

$$\ddot{\vec{r}} = (\ddot{r} - r\dot{\theta}^2)\,\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{\theta}$$

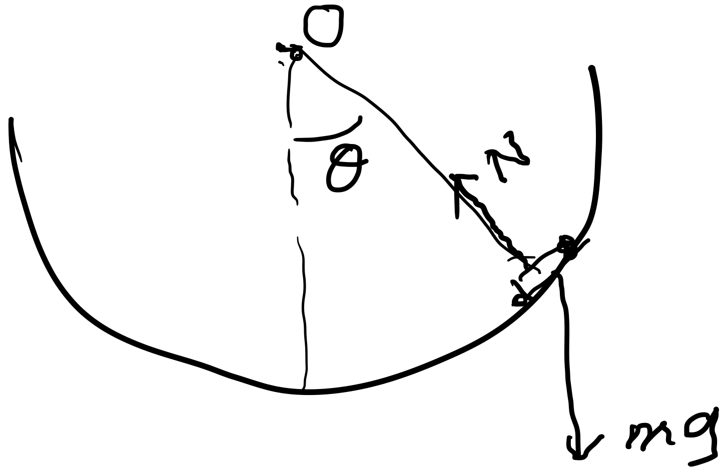
$$\vec{F} = m\vec{a} = m\ddot{\vec{r}}$$

$$\boxed{\begin{aligned} F_r &= m(\ddot{r} - r\dot{\theta}^2) \\ F_\theta &= m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) \end{aligned}}$$

$$F_x = m\ddot{x}$$

$$F_y = m\ddot{y}$$

Oscillating skateboard



skateboard released short way from bottom

how long will it take to come back to same position

$r = R$

$$F_r = m(\ddot{r} - r\dot{\theta}^2) = -mR\dot{\theta}^2$$

$$F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = mR\ddot{\theta}$$

$$F_r = mg\cos\theta - N.$$

$$F_\theta = -mg\sin\theta.$$

$$\boxed{mR\ddot{\theta} = -mg\sin\theta}$$

$$-mg\sin\theta = mR\ddot{\theta}$$

$$\ddot{\theta} = -\frac{g}{R}\sin\theta$$

small angle

$$\boxed{\ddot{\theta} + \frac{g}{R}\theta = 0}$$

$\theta = 0$, $\dot{\theta} = 0$, $\ddot{\theta} = 0$

$\hookrightarrow$ equilibrium position -