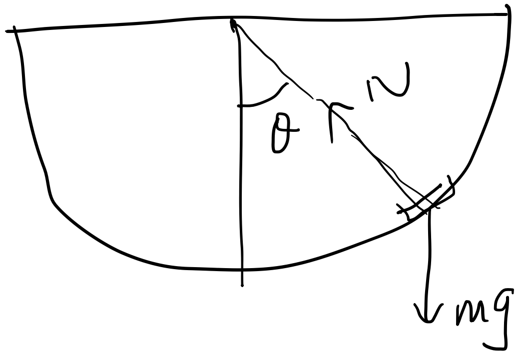
Physics I

Lecture 4

Recap

$$\vec{F} = m\vec{a}$$



$$\ddot{\theta} = -\frac{g}{R}\sin\theta .$$

$$\ddot{\theta} \simeq -\frac{g}{R}\theta .$$

equilibrium position

$$\ddot{\theta} = 0, \quad \theta = 0$$

$$\theta > 0, \quad \ddot{\theta} < 0$$

$$\theta < 0 \quad \ddot{\theta} > 0$$

$$\ddot{\theta} + \frac{g}{R}\theta = 0$$

$$\frac{g}{R} = \omega^2$$

$$\theta(t) = A\sin\omega t + B\cos\omega t$$

$$t=0, \quad \theta = \theta_0, \quad \dot{\theta} = 0 .$$

$$\boxed{\theta(t) = \theta_0 \cos\omega t}$$

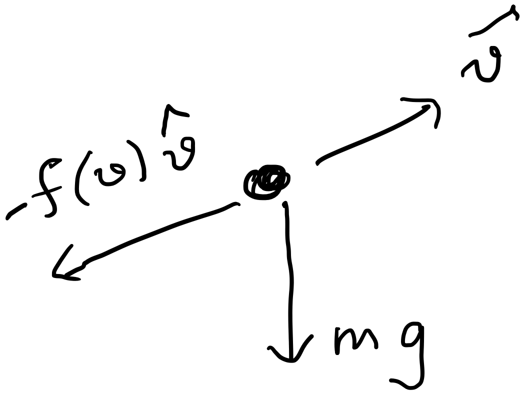
$$\vec{F} = m\vec{a}$$

$$\searrow \vec{F}(v, r, t)$$

Projectile motion with air resistance

Retarding forces

$$\vec{F} = \vec{F}(\vec{v}) = -f(v)\hat{v}$$



At low speeds

$$f(v) = bv + cv^2 = f_{lin} + f_{quad}$$

$f_{lin} \longrightarrow$ viscous drag of medium $\propto$ viscosity of medium
depends on the size of the particle.

Stokes Law $f = 6\pi r \eta v$

$f_{quad} \rightarrow$ projectiles need to accelerate mass of air which they are in contact with; continuously colliding
$\propto$ density of medium and cross sectional area.

for a spherical projectile

$$b = \beta \underbrace{D}_{\text{diameter}} \quad , \quad = \gamma D^2$$

for air $\quad \beta = 1.6 \times 10^{-4} \frac{N.s}{m^2}$

$$\gamma = 0.25 \frac{N.s}{m^2}$$

$$\frac{f_{quad}}{f_{lin}} = \frac{c v^2}{b v} = \frac{\gamma D}{\beta} v = \left(1.6 \times 10^3 \frac{s}{m^2}\right) D v$$

cricket ball vs raindrop

$D = 7cm$ $\quad v = 5 m/s$

$D = 1mm$ $\quad v = 0.6 m/s$.

Milikan oil drop

$D = 1.5 \mu m$

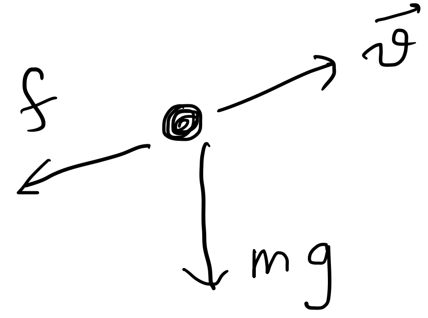
$v = \quad \times 10^{-5} m/s$

$$\frac{f_{quad}}{f_{lin}} \approx 600 \quad \text{cricket ball}$$

$$\frac{f_q}{f_e} \sim 1 \quad \text{raindrop}.$$

$$\frac{f_q}{f_e} \approx 10^{-7} \qquad \vec{f} = -b\vec{v}$$

Linear air resistance



$$\vec{F} = m\vec{g} - b\vec{v} \quad \text{— (1)}$$

$$m\ddot{\vec{r}} = m\vec{g} - b\vec{v} \quad \text{— (2)}$$

$$m\dot{v}_x = -bv_x \quad \text{— (3)}$$

$$m\dot{v}_y = mg - bv_y \quad \text{— (4)}$$

for quadratic drag. $\vec{f} = -cv^2\hat{v} = -cv\vec{v}$

$$m\dot{v}_x = -c\sqrt{v_x^2+v_y^2}\,v_x$$

$$m\dot{v}_y = mg - c\sqrt{v_x^2+v_y^2}\,v_y$$

Horizontal motion with linear drag

at $t = 0$, $x = 0$, $v_x = v_{x0}$.

$$m \dot{v}_x = - b v_x$$

$$\dot{v}_x = - k v_x \qquad \boxed{k = \frac{b}{m}}$$

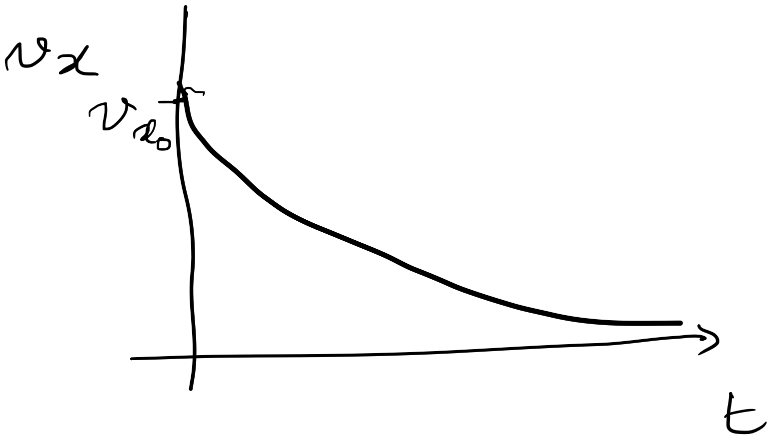
$$\frac{\dot{v}_x}{v} = - k$$

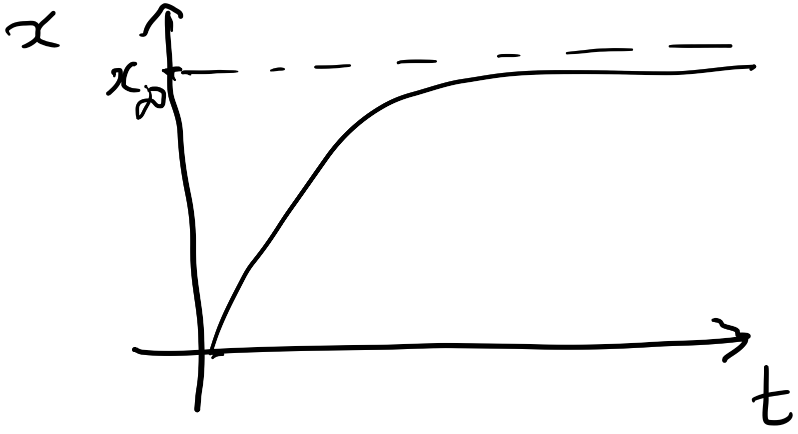
$$\frac{dv}{v} = - k\, dt$$

$$\boxed{v_x = A e^{-kt}} \quad + \text{ initial conditions.}$$

$$\boxed{v_x = v_{x_0} e^{-kt} = v_{x_0} e^{-t/\tau}} \qquad \tau = \frac{1}{k} = \frac{m}{b}.$$

$$v_x(t \to \infty) = 0$$





$$\frac{dx}{dt} = v_{x_0} e^{-kt}$$

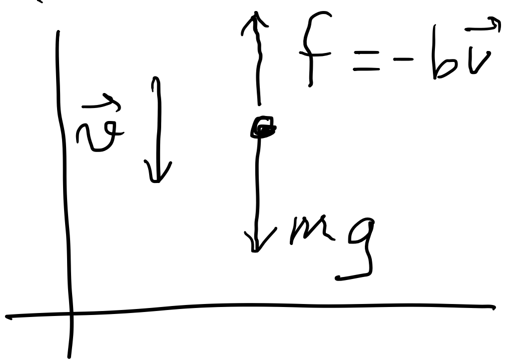
$$x = -\frac{v_{x_0}}{k} e^{-kt} + C \,, \quad t = 0 \,, \; x = 0 \Longrightarrow C = \frac{v_{x_0}}{k}$$

$$\boxed{x = \frac{v_{x_0}}{k}\left(1 - e^{-t/\tau}\right)} \qquad x_\infty = \frac{v_{x_0}}{k}$$

short time limit

$$x \simeq \frac{v_{x_0}}{k}\left(1 - 1 + \frac{t}{\tau}\right) \simeq v_{x_0} t \,.$$

Vertical motion with linear drag



$$m\dot{v}_y = mg - bv_y \quad (v_y > 0)$$

retarding force upward.

$$\boxed{\begin{gathered}\text{when } mg - bv_y = 0 \\ v_y = \frac{mg}{b} = v_{ter}\end{gathered}}$$

$$m\dot{v}_y = -b(v_y - v_{ter})$$

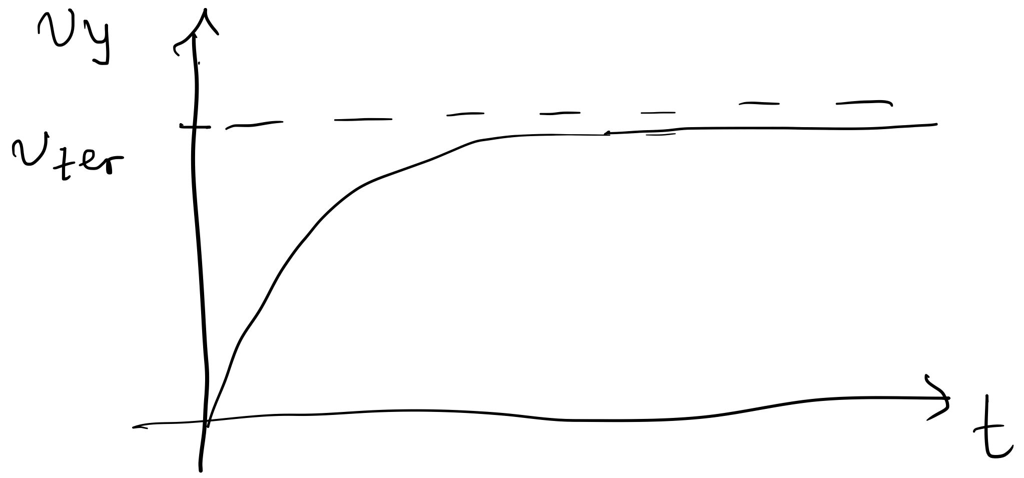
$$\frac{dv_y}{v_y - v_{ter}} = -\frac{b}{m}$$

$$\hookrightarrow \quad v_y - v_{ter} = Ae^{-b/m\,t} \quad ; \quad t = 0,\ v_y = v_{y_0}$$

$$\boxed{A = v_{y_0} - v_{ter}}$$

$$\boxed{\begin{aligned} v_y(t) &= v_{ter} + (v_{y_0} - v_{ter})e^{-t/\tau} \\ &= v_{y_0}e^{-t/\tau} + v_{ter}\left(1 - e^{-t/\tau}\right) \end{aligned}}$$

$t \to \infty$, $v_y(t) \to v_{ter}$



Short time approx.

$v_y(t) \simeq$

$v_{y_0}(1 - t/\tau) + v_{ter}\, t/\tau$.

$\simeq v_{y_0} + (v_{ter} - v_{y_0})\, t/\tau$.

$v_{y_0} = 0$

$v_y(t) \simeq v_{ter}\, t/\tau$.

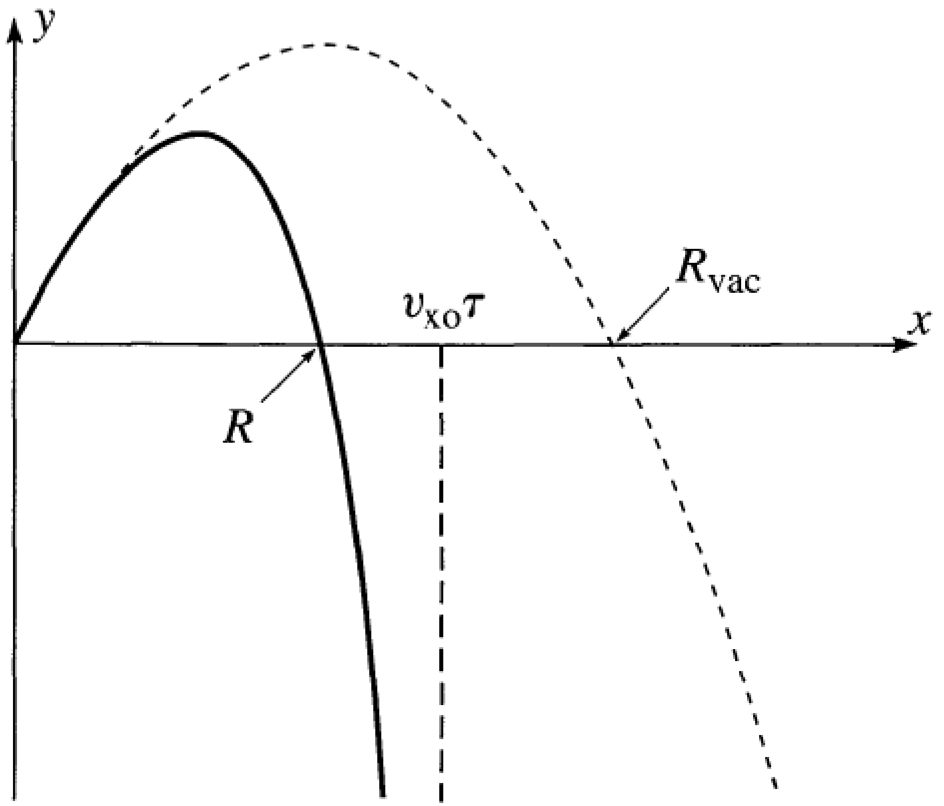
Next integration

$$v_y(t) = v_{ter}\left(1 - e^{-t/\tau}\right)$$

$$\downarrow \quad y(0) = 0$$

$$\boxed{\begin{aligned} y(t) &= v_{ter}\, t + (v_{y_0} - v_{ter})\,\tau\left(1 - e^{-t/\tau}\right) \\ x(t) &= x_\infty\left(1 - e^{-t/\tau}\right) \qquad x_\infty = v_{x_0}\tau \end{aligned}}$$

$\longrightarrow$ orbit of a projectile subject to linear drag.



y
x
$v_{xo}\tau$
R_{vac}
R