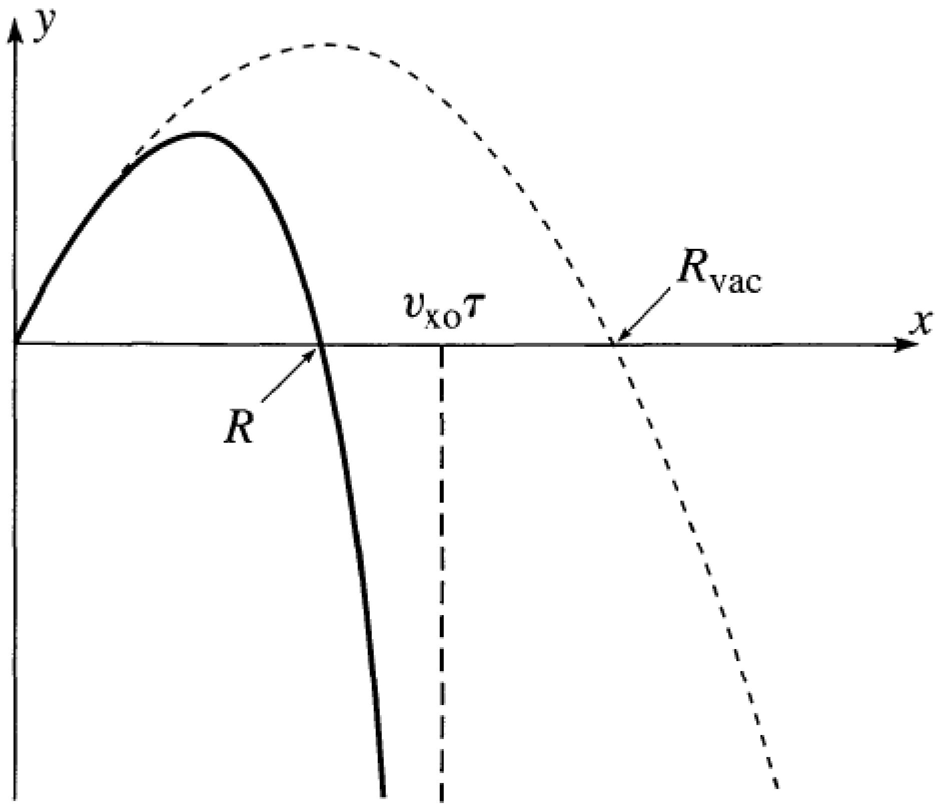
Physics I

Lecture 5



y
x
$v_{xo}\tau$
R_{vac}
R

Trajectory and range in a linear medium

slight change → will take upward y direction as +ve
↳ reverse the sign of v_{ter}

$$x(t) = v_{x_0}\tau\left(1 - e^{-t/\tau}\right) \quad \text{— (1)}$$

$$\boxed{\tau = \frac{m}{b}}$$

$$y(t) = \left(v_{y_0} + v_{ter}\right)\tau\left(1 - e^{-t/\tau}\right) - v_{ter}t \quad \text{— (2)}$$

eliminate t

$$\frac{x}{v_{x_0}\tau} = 1 - e^{-t/\tau}$$

$$\boxed{t = -\tau \ln\left(1 - \frac{x}{v_{x_0}\tau}\right)} \quad \text{— (3)}$$

Plug in 3 into 2

$$y = \frac{(v_{y_0} + v_{ter})x}{v_{x_0}} + v_{ter}\tau \ln\left(1 - \frac{x}{v_{x_0}\tau}\right) \quad (4)$$

limit of small air resistance $\left\{\frac{1}{\tau} = \frac{b}{m}\right)\}$ expand ln to lowest order

$\tau = m/b$

$$\boxed{y \simeq -\frac{1}{2} g \frac{x^2}{v_{x_0}^2}}$$ vacuum case.

$v_{ter}, \tau \rightarrow \infty$, vacuum case.

$x \rightarrow v_{x_0}\tau$, $y \rightarrow -\infty$ vertical asymptote

Horizontal Range

Recall $\boxed{R_{vac} = \dfrac{2 v_{x0} v_{y0}}{g}}$ case $b = 0$

Range: x for $y = 0$

$$y = \frac{(v_{y0} + v_{ter})x}{v_{x0}} + v_{ter}\tau \ln\left(1 - \frac{x}{v_{x0}\tau}\right)$$

$$0 = \frac{(v_{y0} + v_{ter})R}{v_{x0}} + v_{ter}\tau \ln\left(1 - \underbrace{\frac{R}{v_{x0}\tau}}_{\text{small for } b \text{ small}}\right)$$

$$\ln(1-\varepsilon) = -\left(\varepsilon + \frac{1}{2}\varepsilon^2 + \frac{1}{3}\varepsilon^3 + \cdots\right).$$

large τ

$$\left(\frac{v_{y_0} + v_{ter}}{v_{x_0}}\right) R - v_{ter}\tau \left[\frac{R}{v_{x_0}\tau} + \frac{1}{2}\left(\frac{R}{v_{x_0}\tau}\right)^2 + \frac{1}{3}\left(\frac{R}{v_{x_0}\tau}\right)^3 + \cdots\right]$$

$$= 0$$

one trivial soln: $R = 0$

$$\boxed{\frac{v_{y_0}}{v_{x_0}} - \frac{v_{ter} R}{2 v_{x_0}^2 \tau} - \frac{1}{3}\frac{R^2}{v_{x_0}^3 \tau^2} = 0}$$

$$\frac{v_{x_0}}{v_{y_0}} - \frac{v_{ter}}{2}\frac{R}{v_{x_0}^2 \tau} - \frac{1}{3}\frac{R^2 v_{ter}}{v_{x_0}^2 \tau^2} = 0 .$$

$$R \cong \frac{2 v_{x_0} v_{y_0}}{g} - \underbrace{\frac{2}{3 v_{x_0} \tau}}_{\rightarrow \text{ small}} R^2$$

$$\boxed{\frac{v_{ter}}{\tau} = g}$$

R at best R_{vac} .

first approx

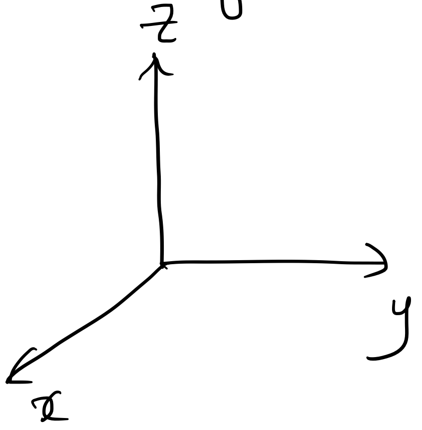
$$R \simeq \frac{2 v_{x_0} v_{y_0}}{g}$$

2nd approx

$$\boxed{R \simeq \frac{2 v_{x_0} v_{y_0}}{g} - \frac{2}{3 v_{x_0} \tau} R_{vac}^2 \cong R_{vac}\left(1 - \frac{4}{3}\frac{v_{y_0}}{v_{ter}}\right)}$$

Another example of a velocity dependent force.

Charge in a uniform magnetic field.



$$\vec{B} = B_0\,\hat{y}$$

$$\vec{F}_{mag} = q\left(\vec{v} \times \vec{B}\right)$$

$$\vec{v} = \dot{x}\,\hat{x} + \dot{y}\,\hat{y} + \dot{z}\,\hat{z}$$

$$\vec{a} = \ddot{x}\,\hat{x} + \ddot{y}\,\hat{y} + \ddot{z}\,\hat{z}$$

Eqn. of motion $\vec{F} = m\vec{a}$

$$\boxed{\begin{aligned} m\ddot{x} &= \left(\vec{F}_{mag}\right)_x = -qB_0\dot{z} \\ m\ddot{y} &= 0 \\ m\ddot{z} &= qB_0\dot{x} \end{aligned}}$$

$$y = \dot{y}_0 t + y_0 \quad \text{— (2)}$$

$$y_0 = y(t=0)$$
$$\dot{y}_0 = \dot{y}(t=0)$$

$$\left.\begin{aligned} m\ddot{x} &= -qB_0\dot{z} \\ m\ddot{z} &= \cancel{-}\, qB_0\dot{x} \end{aligned}\right\} \quad \begin{aligned} &\text{— (1)} \\ &\text{— (3)} \end{aligned}$$

$$\alpha = \frac{qB_0}{m}$$

$$\left.\begin{aligned} \ddot{x} &= -\alpha\dot{z} \\ \ddot{z} &= \alpha\dot{x} \end{aligned}\right\}$$

Take derivative

$$\left.\begin{aligned} \dddot{x} &= -\alpha\ddot{z} = -\alpha^2\dot{x} \\ \dddot{z} &= +\alpha\ddot{x} = -\alpha^2\dot{z} \end{aligned}\right\}$$

$\dot{z} = u, \quad \dot{x} = v$

$$\boxed{\ddot{u} = -\alpha^2 u, \quad \ddot{v} = -\alpha^2 v}$$

$\ddot{u} + \alpha^2 u = 0$

$\ddot{v} + \alpha^2 v = 0$

$\rightarrow \dot{z} = u = \tilde{A} \sin\alpha t + \tilde{B} \cos\alpha t.$

Integrate once

$z = -\frac{\tilde{A}}{\alpha} \cos\alpha t + \frac{\tilde{B}}{\alpha} \sin\alpha t$

$$\boxed{z(t) = A' \cos\alpha t + B' \sin\alpha t + z_0}$$

Similarly

$$\boxed{x = A \cos\alpha t + B \sin\alpha t + x_0.}$$

Are A, B, A', B' all independent

$$\ddot{x} = -\alpha \dot{z}$$

$$\ddot{z} = \alpha \dot{x}$$

$$-\alpha^2 A\cos\alpha t - \alpha^2 B\sin\alpha t = -\alpha(-\alpha A'\sin\alpha t + \alpha B'\cos\alpha t)$$

valid for all t, $t = 0$, $t = \pi/2\alpha$.

$$-\alpha^2 A = -\alpha^2 B' \qquad -\alpha^2 B = \alpha^2 A'$$

$$\boxed{A = B' \qquad B = A'}$$

$$(x - x_0) = A\cos\alpha t + B\sin\alpha t$$

$$(y - y_0) = \dot{y}_0 t$$

$$(z - z_0) = -B\cos\alpha t + A\sin\alpha t.$$

$t = 0$ $\quad \dot{z} = \dot{z}_0$, and $\dot{x} = 0$.

$B = 0$, $\alpha A = \dot{z}_0$

$$\boxed{\begin{aligned} x - x_0 &= \frac{\dot{z}_0}{\alpha}\cos\alpha t \\ (y - y_0) &= \dot{y}_0 t \\ (z - z_0) &= \frac{\dot{z}_0}{\alpha}\sin\alpha t \end{aligned}}$$

what trajectory is this ?.

$$(x - x_0)^2 + (z - z_0)^2$$

$$= \left(\frac{\dot{z}_0^2 m^2}{q^2 B_0^2}\right)$$

right circular helix.