Physics I

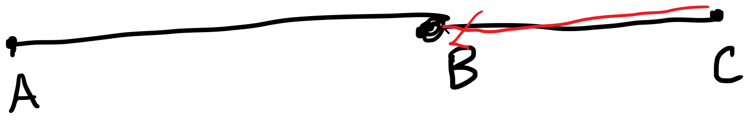
Lecture 8

One dimensional linear motion

• $F = F(x)$: satisfies the 1st condition for conservative force

↳ 2nd condition $W_{12} = \int_1^2 F(x)\,dx$ is path independant

↳ Is automatically satisfied in 1D.



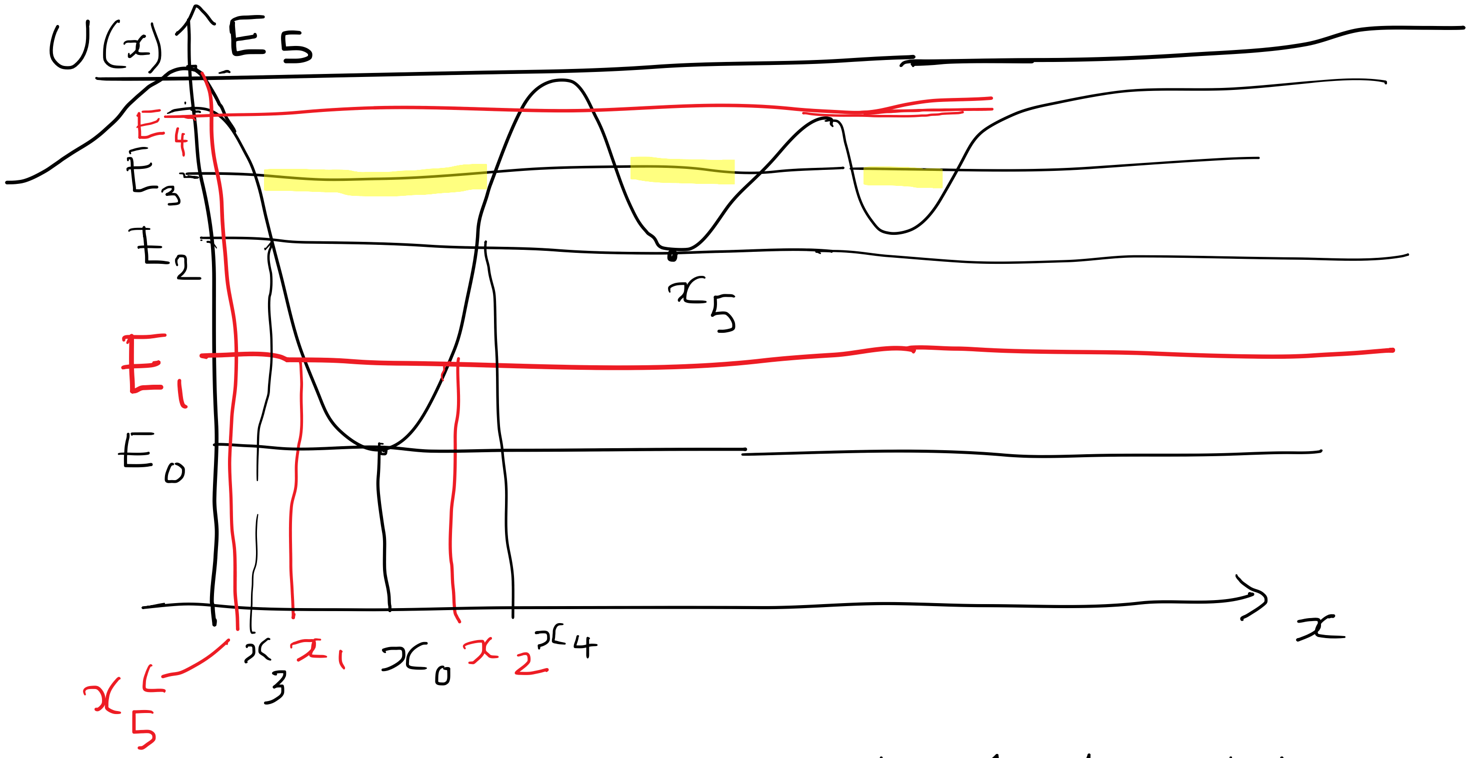
$$W_{ABCB} = W_{AB} + \underbrace{W_{BC} + W_{CB}}_{=0}$$

If $F = F(x) \longrightarrow U(x)$ exists $E = T + U = \text{const}$.

- We can learn great deal about the motion by looking at graph of $U(x)$ without explicitly obtaining solution.

- Obs. 1 $T \geqslant 0$. $\therefore$ for a given energy E, the motion will be confined to regions of the x-axis where

$\{E = T + U\}$ $\boxed{U(x) \leqslant E}$ $\longrightarrow$ classically allowed region.



[$E < E_0 \Rightarrow T < 0$ in response to question in class]

{ $E = E_0 = T + U$
$= T + U_0$
$T = 0$ }

- With energy E_0, particle will just sit at x_0

- With energy E_1, classically allowed region $x_1 \leq x \leq x_2$
  x_1, x_2 called turning points, $\boxed{U(x) = E_1}$, the motion must be bounded, oscillatory.

- With energy E_2, classically allowed regions are $x_3 \leq x \leq x_4$
  $x = x_5$; either oscillate between x_3 & x_4 turning pts or sit at x_5

- With energy E_5, there is only <u>one</u> turning point, particle comes in from ∞ hits barrier/turning pt and goes back to ∞ along x-axis. speeding up over the valleys and slowing down ~~over the~~ at the hill. Unbounded motion.

- With energy $> E_5$ no turning points and particle moves in one direction only modulating the speed according to the depth of the potential

x_0 : Stable equilibrium position

$$U(x) = \underbrace{U(x_0)}_{\text{redefine ref pt.}} + \underbrace{\left(\frac{dU}{dx}\right)_{x_0}}_{0} (x - x_0) + \frac{1}{2}\left(\frac{d^2U}{dx^2}\right)_{x_0} (x - x_0)^2 + \cdots$$

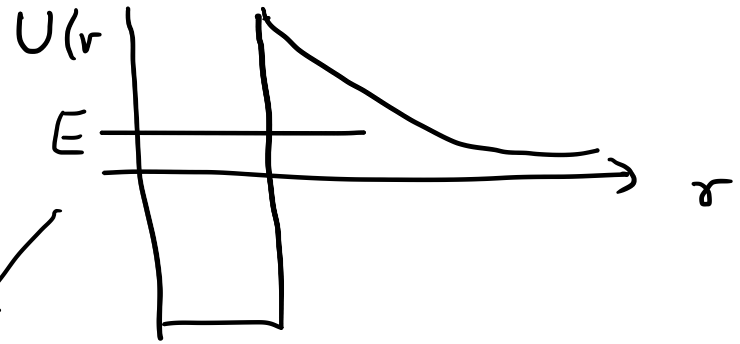
$$U(x) \simeq \frac{1}{2} k (x - x_0)^2$$

$$\left(\frac{dU}{dx}\right)_{x_0} = 0 \quad , \quad \left(\frac{d^2U}{dx^2}\right)_{x_0} \geq 0 \Rightarrow \text{stable.}$$

$$\left(\frac{dU}{dx^2}\right)_{x_0} \leq 0 \Rightarrow \text{unstable.}$$

Why "classically" allowed ?

α-decay



nuclear potential.

in quantum mech.

$U(x) \leq E$ condn violated.

energy of α-particles

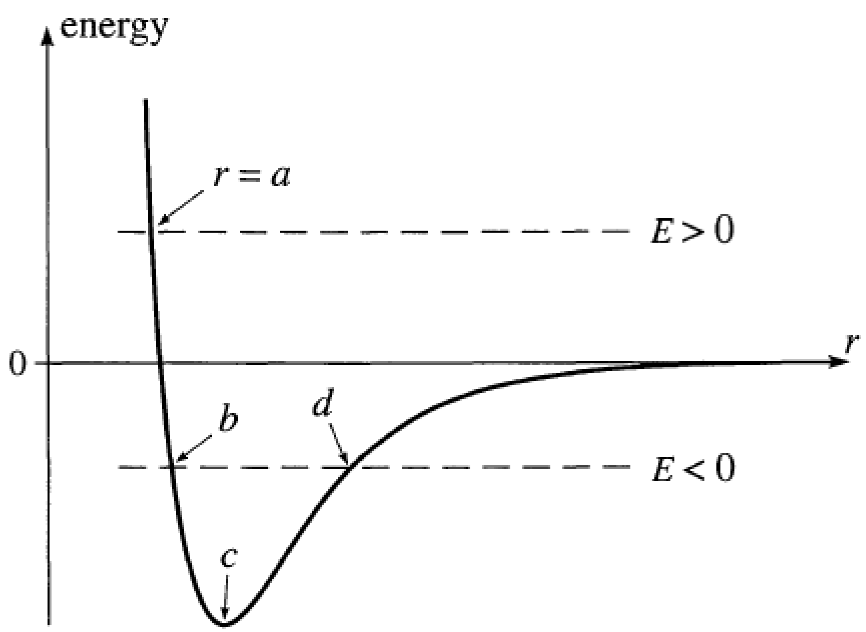


Figure 4.12 The potential energy for a typical diatomic molecule such as HCl, plotted as a function of the distance r between the two atoms. If $E > 0$, the two atoms cannot approach closer than the turning point $r = a$, but they can move apart to infinity. If $E < 0$, they are trapped between the turning points at b and d and form a bound molecule. The equilibrium separation is $r = c$.

- One-dimension motion can be completely solved in principle.

$$E = \frac{1}{2} m v^2 + U(x)$$

$$v = \frac{dx}{dt} = \pm \sqrt{\frac{2}{m}\left(E - U(x)\right)}$$

$$x(t) = ?$$

integrate

$$\boxed{t - t_0 = \pm \sqrt{\frac{m}{2}} \int_{x_0}^{x} \left[E - U(x')\right]^{-1/2} dx'}$$

$\rightarrow$ complete soln.

two initial cond E, x_0.