Physics I
ISI B.Math
HW set 2
Marks = 40

1. A particle P of unit mass moves on the positive x- axis under the force field

$$F = \frac{36}{x^3} - \frac{9}{x^2}$$

where $x > 0$.
Show that the motion of P consists of either(i) periodic oscillation between two extreme points or (ii) an unbounded motion with one extreme point, depending upon the value of total energy. Initially, P is projected from the point $x = 4$ with speed 0.5. Show that P oscillates between two extreme points and find the period of the motion. You may make use of the formula

$$\int_a^b \frac{xdx}{[(x-a)(b-x)]^{\frac{1}{2}}} = \frac{\pi(a+b)}{2}$$

Show that there is a single equilibrium position for P and that it is stable. Find the period of small oscillations about this point. (2+ 4 + 4 = 10)

2. A particle is under the influence of a force $F = -kx + \frac{kx^3}{a^2}$, where k and α are constants and k is positive. Determine $U(x)$ and discuss the motion. What happens when the total energy $E = \frac{1}{4}k\alpha^2$? (10)

3. A particle moves towards $x = 0$ under the influence of a potential $V(x) = -A|x|^n$, where $A > 0$ and $n > 0$. The particle has barely enough energy to reach $x = 0$. For what values of n will it reach $x = 0$ in finite time? (5)

4. Which of the following forces are conservative ? If conservative, find the potential energy $U(\mathbf{r})$. (5 + 5 + 5 =15)
(a) $F_x = ayz + bx + c, F_y = axz + bz, F_z = axy + by$
(b) $F_x = -ze^{-x}, F_y = \ln z, F_z = e^{-x} + \frac{y}{z}$
(c) $\mathbf{F} = \frac{a}{r}\hat{\mathbf{r}}$ (a, b, c) are constants