Statistics-III – Assignment 1

1. (a) Suppose X and Y are i.i.d $N(0,1)$. Using the Jacobian method, show that $\frac{X}{Y}$ has the Cauchy distribution.
(b) If X and Y are independent continuous random variables which are symmetric about 0, then $\frac{X}{Y}$ and $\frac{X}{|Y|}$ have the same distribution. Therefore, if X and Y are i.i.d $N(0,1)$, $\frac{X}{|Y|}$ also has the Cauchy distribution.

2. Suppose X and Y are i.i.d $N(0,1)$. Consider the transformation $(X,Y) \to (R,\Theta)$ where $X = R\cos\Theta$ and $Y = R\sin\Theta$. Find the joint distribution of (R,Θ).

3. Let $Y_1,\ldots,Y_n$ be independent random variables with unit variance, and let $X_1 = Y_1$, $X_i = Y_i - Y_{i-1}$ for $1 < i \le n$. Find the covariance matrix of $\mathbf{X} = (X_1, X_2, \ldots, X_n)'$.

4. Suppose $\Sigma =$ Cov $(X) = \begin{pmatrix} 1 & \rho & \rho \\ \rho & 1 & \rho \\ \rho & \rho & 1 \end{pmatrix}$. Show that $-1/2 \le \rho \le 1$.

5. Let $X_1,\ldots,X_n$ be i.i.d Exponential with mean 1. Define $Y_1 = nX_{(1)}$, $Y_2 = (n-1)(X_{(2)} - X_{(1)})$, $Y_i = (n-i+1)(X_{(i)} - X_{(i-1)})$ for $3 \le i \le n$, where $X_{(1)} \le X_{(2)} \le \cdots \le X_{(n)}$ are the order statistics. Show that $Y_1,\ldots,Y_n$ are i.i.d Exponential with mean 1.
(Hint. Note, in the joint density, $\sum_{i=1}^n x_{(i)} = \sum_{i=1}^n y_i$.)