Statistics-III – Assignment 4

1. If C is a generalized inverse of $X'X$ prove the following.
(a) C' is also a generalized inverse of $X'X$.
(b) A symmetric generalized inverse of $X'X$ exists.
(c) CX' is a generalized inverse of X.
(d) XCX' is unique.
(e) XCX' is symmetric and idempotent.
(f) Column spaces of XCX' and X are the same.

2. Consider the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ -1 & 1 & -3 \\ 1 & 2 & 0 \end{pmatrix}$.
(a) Find a generalized inverse $(A'A)^-$ of $A'A$.
(b) Find a generalized inverse $(AA')^-$ of AA'.

3. (a) For all matrices $A_{m\times n}$, is it true that if B is a g-inverse of A, then A is a g-inverse of B?
(b) Let $A = \begin{pmatrix} B & 0 \\ 0 & C \end{pmatrix}$, where B is $r_1 \times s_1$ and C is $r_2 \times s_2$. Let B^- and C^- be any g-inverses of B and C respectively. Show then that $G = \begin{pmatrix} B^- & 0 \\ 0 & C^- \end{pmatrix}$ is a generalized inverse of A. Must all g-inverses of A have the form G?
(c) Find a generalized inverse of $A = \begin{pmatrix} \mathbf{1}_3\mathbf{1}'_3 & 0 \\ 0 & 2\mathbf{1}_2\mathbf{1}'_2 \end{pmatrix}$, where $\mathbf{1}_k$ is the k-vector $(1, 1, \ldots, 1)'$.