

### Statistics-III – Assignment 5

1. If columns of  $X$  are linearly dependent in the Gauss-Markov model, prove that there is no matrix  $C$  such that  $C\mathbf{Y}$  is an unbiased estimator of  $\beta$ .
2. Prove that all linear functions  $\mathbf{a}'\beta$  are estimable in the Gauss-Markov model iff the columns of  $X$  are linearly independent.
3. If  $\mathbf{a}'_1\beta, \mathbf{a}'_2\beta, \dots, \mathbf{a}'_l\beta$  are estimable, prove that any linear combination of these is also estimable (in the Gauss-Markov model).
4. Prove that in the Gauss-Markov model  $\mathbf{a}'\beta$  is estimable if and only if  $\mathbf{a}'(X'X)^{-1}X'X = \mathbf{a}'$ .