Statistics-III – Assignment 7

1. Consider testing for the usefulness of the regressors in a linear model, $Y = X\beta + \epsilon$, $\epsilon \sim N_n(0, \sigma^2 I_n)$ with $H_0 : \beta_1 = \beta_2 = \cdots = \beta_{p-1} = 0$, where $X_{n\times p}$ has rank $r \leq p$. Let R^2 denote the coefficient of determination. Find the probability distribution of R^2 under H_0.

2. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$. Show that $E\left(\Phi\left(\frac{X_{(i)}-\mu}{\sigma}\right)\right) = \frac{i}{n+1}$, where $X_{(i)}$ is the ith order statistic.

3. Show that

$$\rho_{ij.kl} = \frac{\rho_{ij.k} - \rho_{il.k}\rho_{jl.k}}{\sqrt{\left(1-\rho^2_{il.k}\right)\left(1-\rho^2_{jl.k}\right)}}.$$