Topology

Assignment - 1

1. + A metric space is called <u>sequentially compact</u> if every sequence has a convergent subsequence.

   + Call a metric space <u>totally bounded</u> if for every $\epsilon > 0$, the metric space can be covered by finitely many open balls of radius ϵ.

   Prove that TFAE for a metric space (X, d):

   (i) X is compact.

   (ii) X has the Bolzano-Weirstrass property, i.e. every infinite set has a limit point in X.

   (iii) X is sequentially compact.

   (iv) X is totally bounded and complete. (2)

2. Given an example to show Heine-Borel theorem fails in metric spaces, i.e. a set may be closed and bounded yet fail to be compact. (1)

3. Prove that a totally bounded metric space is <u>separable</u> i.e. contains a countable dense subset. (1)

——— x ——— x ——— x ———