Homework-Assignment (4)

1. Let X be a topological space, $A \subseteq X$. Consider the quotient space X/A. Let $[A]$ be the point in X/A corresponding to A. If A is either open or closed, Show that $X - A$ is homeomorphic to $X/A - \{[A]\}$.

2. Let X be a top. space and $\sim$ be the equivalence relation on X corresponding to the connected components as its equivalence classes. Prove that the quotient $X/\sim$ is totally disconnected.

3. Let $p: X \to Y$ be a quotient map. Assume that all fibers of p are connected. Show that an open (or closed) subset $F \subseteq Y$ is connected $\Leftrightarrow$ $p^{-1}(F)$ is connected.

4. $p: X \to Y$ be a quotient map & X be locally connected. Prove that Y too is locally connected.

5. Let X be a top space, $\sim$ an equivalence on X, and let $X/\sim$ have the quotient topology, $p: X \to X/\sim$ the quotient map. Then p is open (closed) $\Leftrightarrow$ $\bigcup_{u \in U} [u]$ is open (closed) for every open (closed) subset $U \subseteq X$. (2)

6. Let X be regular and $A \subseteq X$ be closed then Prove that X/A is Hausdorff. (2)

7. Let X be normal, $A \subseteq X$ closed, then prove that X/A is normal.

* Problems 5 and 6 constitute Assignment-4. Submit by March-31.