Homework - 4

1. Recall that a top. space X is <u>locally</u> <u>connected</u> if every point has a local basis of connected open sets. Is it true that a connected subspace of a locally connected space is locally connected? Explain.
2. Give an example to show that local connectedness may not be preserved under continuous functions.
3. A finite non-empty product of locally connected spaces is locally connected, while an arbitrary non-empty product of loc. conn. space may not be.
4. Any arbitrary non-empty product of connected, locally conn. spaces is locally connected.
5. A top. space X is locally connected <u>if</u> the conn. components of every open subspace are open in X. Converse?
6. Prove Urysohn's lemma using Tietze's ext^n thm!
7. State and prove a generalization of Tietze's ext^n theorem which relates to functions taking values in $\mathbb{R}^n$.
8. Let X be completely regular, then the weak topology generated by $\mathcal{C}(X, \mathbb{R})$ equals the given ~~topology~~ on X.

9. Let $X \neq \phi$ a set & $\{X_i\}_i$ a non-empty class of top. spaces. For each i, $f_i : X \longrightarrow X_i$ be given. Let τ be the weak topology on X gen. by the f_i's.

(a) Show that τ equals the topology on X gen. by the class of all inverse images in X of open sets in the X_i's.

(b) Let Y be a subspace of (X, τ), τ as in (a). Show that the subspace top. on Y is the weak top. gen. by the restrictions $f_i|_Y$.

10. Give a complete description of the weak topo. on $\mathbb{R}$ gen. by the family $\{f_i\}_i$ of <u>all</u> continuous maps: $\mathbb{R} \longrightarrow \mathbb{R}$, wrt the usual top. on $\mathbb{R}$.

11. Locally path connected spaces are defined exactly analogously as (1). Prove the analogue of (5) in this context.
Prove that for a locally path conn. space connected components are same as path comps.

12. Let X be a connected normal space having more than one point. Show that X is uncountable.

13. A connected & regular space having at least two points is uncountable.
