Homework

1. Let $V_{n,k}$ be the set of k-frames in $\mathbb{R}^n$; a k-frame is an orthonormal set of k vectors in $\mathbb{R}^n$. Prove that $O(n)$ acts transitively on $V_{n,k}$. Hence identify $O(n)/O(n-1) \approx V_{n,k}$.

   ($V_{n,k}$ is called the Stiefel manifold of k-frames).

2. Prove in detail $SU(n)/SU(n-1) \simeq S^{2n-1}$, $n \geq 2$.

3. $\phi: G \to H$ be a surjective continuous homomorphism of topological groups. Let $K := \mathrm{Ker}(\phi)$. Show that K is closed and normal (assume these groups are T_1). If G is compact, prove that G/K is homeomorphic to H by a group isomorphism.

4. Show that $Sp(1) \xrightarrow{\sim} S^3$.

5. Prove $SO(n)$ is connected.

6. Prove $U(n)/SU(n) \xrightarrow{\sim} S^1$.

7. Prove that $\mathbb{RP}^n \xrightarrow{\sim} O(n+1)/(O(n)\times O(1))$,

   $\mathbb{CP}^n \xrightarrow{\sim} U(n+1)/(U(n)\times U(1))$, $\mathbb{HP}^n \xrightarrow{\sim} ??$

8. Prove analogues of (1) for other groups.