Indian Statistical Institute, Bangalore
B. Math.
Third Year, First Semester
Analysis on Graphs
Home Assignment II   Due Date : October 25, 2023
Instructor: B V Rajarama Bhat

(1) Let $K_{p,q}$ and $K_{r,s}$ be two complete bipartite graphs with p, q, r, s denoting the number of vertices. (i) When are they co-spectral? The normalized Laplacian of a graph G is defined as
$$D^{-\frac{1}{2}}(D-A)D^{-\frac{1}{2}},$$
where D is the degree matrix and A is the adjacency matrix. (ii) When are $K_{p,q}$ and $K_{r,s}$ co-spectral with respect to the normalized Laplacian?

(2) Let S_3 be the group of permutations of $\{1, 2, 3\}$. Obtain two different graphs as Cayley graphs of S_3.

(3) (Line graph). Let G be a graph with at least one edge. Then the line graph of G is the graph $\hat{G}$, where $V(\hat{G}) = E(G)$ (So the vertex set of $\hat{G}$ is the set of edges of G) and two edges of G form an edge in $\hat{G}$ if the edges have a common incident vertex. (i) If N is the incidence matrix of G, show that $N^t N - 2I$ is the adjacency matrix of $\hat{G}$. (ii) If λ is an eigenvalue of the adjacency matrix of $\hat{G}$, then $\lambda \geq -2$. (iii) Is this true of false: $\hat{\hat{G}} = G$. (Justify your claim).

(4) Let G be a graph with n vertices. Suppose λ_1 is the maximum eigenvalue of the Laplacian of G. Show that $\lambda_1 \leq n$.

(5) Show that the Seidel adjacency matrix S of a graph with n vertices has rank $(n-1)$ or n. (Hint: $\det S \equiv (n-1)(\text{mod}2)$.)