Assignment-1

1. Let γ be a unit speed plane curve, κ_s be its (signed) curvature. Assume κ_s is nowhere zero. Define the centre of curvature $\epsilon(s)$ of γ at $\gamma(s)$ by $\epsilon(s) = \gamma(s) + \frac{1}{\kappa_s(s)} n_s(s)$,

   Where n_s is the (signed) unit normal of γ.

   Prove that the circle with centre $\epsilon(s)$ and radius $\left|1/\kappa_s(s)\right|$ is tangent to γ at $\gamma(s)$ and has the same curvature as γ at $\gamma(s)$. This circle is called the <u>osculating circle to γ at $\gamma(s)$</u>. [2]

2. (i) Let γ be a curve of general type in $\mathbb{R}^n$; $\{t_1, \dots, t_n\}$ be its distinguished Frenet frame & $0 \leq k \leq n$. Recall that $\gamma^{(k)} = c_1 t_1 + \dots + c_k t_k$ for suitable functions $c_1, \dots, c_k$. Prove that

   $$c_k = |\dot{\gamma}|^k \kappa_1 \cdots \kappa_{k-1} ,$$

   Where κ_i's are the curvatures of γ. [1]

   (ii) Compute the curvatures of the "moment" curve $\gamma(t) = (t, t^2, \dots, t^n)$ at $t = 0$. [1]