Assignment-3

1. Let M, N be smooth manifolds, $f: M \to N$ a smooth map. Let $\Gamma(f) \subseteq M \times N$ be the graph of f. Is $\Gamma(f)$ a submanifold of $M \times N$? Explain.

(2)

2. Let M, N be smooth manifolds, $f: M \to N$ be smooth having the property:
for every $p \in M$, $\exists$ a neighbourhood U in M and a chart (V, y) around $f(p)$ in N such that with $x_i := y_i \circ f|_U$, $1 \leq i \leq m = \dim M$, as coordinate functions, (U, x) is a chart at p. Here $y_1, y_2, \cdots, y_n$ are the coordinate functions on V, $n = \dim N$; In particular, $m \leq n$.
Prove that f is an immersion.

(2)