Homework-1

1. Describe the Weingarten map of a plane curve (assume regularity).

2. (i) Let $f(x,y)$ be a smooth function $: \mathbb{R}^2 \to \mathbb{R}$ and assume that at every point of the "level curve"
$$\mathcal{C} = \{(x,y) \in \mathbb{R}^2 \mid f(x,y) = 0\},$$
at least one of $\frac{\partial f}{\partial x}$ & $\frac{\partial f}{\partial y}$ ~~are~~ is non zero. Let $p = (x_0, y_0) \in \mathcal{C}$. Then $\exists$ a regular parameterized curve $\gamma(t)$, defined on an open interval around 0, such that γ passes through p at $t=0$ and $\gamma(t) \in \mathcal{C}$ $\forall t$.

(ii) Let γ be a regular parameterized plane curve & $\gamma(t_0) = (x_0, y_0)$. Then $\exists$ a smooth map $f(x,y)$, defined for x, y in some open intervals containing x_0 & y_0 respectively, and the condition on $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ as above is satisfied, such that $\gamma(t) \in \mathcal{C} = \{(x,y) \mid f(x,y) = 0\}$ $\forall t$ in some open interval containing t_0.

3. Let P, Q be points in $\mathbb{R}^n$. Prove that the line segment joining P and Q has the shortest length among all parameterized curves joining P to Q.