Homework-4

1. Let M be param. hypersurface in $\mathbb{R}^3$, $p \in M$. Prove that for $v, w \in T_pM$, $L_p(v) \times L_p(w)$

$$= K(p)\, v \times w$$, K the Gaussian

curvature.

2. Suppose the principal curvatures of a (connected Ω) param. hypersurface : $\Omega \to \mathbb{R}^3$ vanish. Show that the hypersurface is part of a plane.

3. Recall, the mean curvature of param hypersurface $M \subseteq \mathbb{R}^n$ is $H(p) = tr(L_p)/(n-1)$ L_p the Weingarten map (same not^n as (1)). For M as above, $M \subseteq \mathbb{R}^3$, show that

$$H^2 \geq K.$$ What are the points

Where $H = K$ holds?

4. Determine the Weingarten map for the param sphere S^n of radius r, Write $H(p)$ for $p \in S^n \subseteq \mathbb{R}^{n+1}$

5. Let M, N be manifolds of dim m & n resp. Prove (i.e. give an atlas) that $M \times N$ is a manifold of dim $(m+n)$.

6. Let $\mathcal{F}$ denote the set of all <u>flags</u> of subspaces $V_1 \subset V_2 \subset \cdots \subset V_{n-1} \subset V_n = V$, $\dim V_i = i$. Give (?) a manifold structure to $\mathcal{F}$.

x ——— x ——— x