Assignment-1

1. Let $U \subseteq \mathbb{R}^n$ be open, $f: U \to \mathbb{R}^n$ be a continuously differentiable injective function such that $Df(x)$ is invertible $\forall\, x \in U$. Show that $f(U)$ is open and $f^{-1}: f(U) \to U$ is differentiable. Prove that $f(V)$ is open for any open set $V \subseteq U$. (1)

(Continuously differentiable $\equiv$ diff. at all points of U and $\partial f_i / \partial x_j$ are all continuous.)

2. Let $f: \mathbb{R} \to \mathbb{R}$ be a C^r-local diffeomorphism at each point, $r \geq 1$; then prove that f is a global C^r-diffeomooplism. (1)