# Assignment - 2

1. Prove that a map $X : M \to \bigcup_p T_pM$ with $X(p) \in T_pM$ $\forall\, p \in M$, defines a smooth vector field on a smooth manifold $M$ if and only if $\forall\, f \in C^\infty(M)$, the map $X_f : M \to \mathbb{R}$ given by $X_f(p) := \cancel{X_p(f)}\ X(p)(f)$ is smooth. (1)

2. Let $M$ be a smooth manifold, $p \in M$ and $v_0 \in T_p(M)$ be fixed. Prove that $\exists$ a smooth vector field $X$ on $M$ such that $\forall\, q \in M$, $X(q)$ is defined and $X(p) = v_0$; hence any tangent vector can be 'extended' to a vector field on $M$. (1).