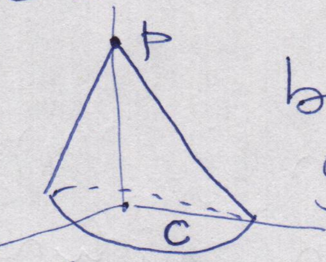
Homework-2

1. Let S^n be the unit sphere in $\mathbb{R}^{n+1}$ with the subspace topology, $N = (0,1) \in \mathbb{R}^n \times \mathbb{R}$ & $S = (0,-1) \in \mathbb{R}^n \times \mathbb{R}$ be the north & south poles respectively. Let $U_N = S^n \setminus \{N\}$ & $U_S = S^n \setminus \{S\}$ and $\varphi_N : U_N \to \mathbb{R}^n$, $\varphi_S : U_S \to \mathbb{R}^n$ be given by $\varphi_N((a_1, \ldots, a_{n+1})) = \frac{1}{1-a_{n+1}} (a_1, \ldots, a_n)$ and $\varphi_S((a_1, \ldots, a_{n+1})) = \frac{1}{1+a_{n+1}} (a_1, \ldots, a_n)$. Describe the transition maps $\varphi_S^{-1} \circ \varphi_N$, $\varphi_N \circ \varphi_S^{-1}$ and check the smoothness of S^n with the atlas $\mathcal{A} = \{(U_N, \varphi_N), (U_S, \varphi_S)\}$.

2. Let $\mathcal{A} = \{(U_i, \varphi_i), 1 \leq i \leq n+1\}$ be the atlas of $\mathbb{RP}^n$ as defined in Lecture-6. Describe the transition maps for $\mathcal{A}$ and check smoothness of $(\mathbb{RP}^n, \mathcal{A})$.

3. Let $X =$ 

be the cone in $\mathbb{R}^3$ given suitably, with the boundary circle removed. Can we give an atlas for X so that it is a smooth manifold? Explain.