Homework-4

~~To be submitted~~

1. Let M be a smooth manifold, $f, g \in C^{\infty}(M)$ be such that on some open subset $V \subseteq M$, $f|_V = g|_V$ (i.e. $f(x) = g(x)\ \forall x \in V$). Prove that $X(f) = X(g)$.

2. Recall: We have charts on $\mathbb{RP}^2$ given by

$$[(x,y,z)] \longmapsto (u_1, u_2) = (x/z, y/z) \text{ on } U_3 = \{z \neq 0\},$$

$$[(x,y,z)] \longmapsto (v_1, v_2) = (x/y, z/y) \text{ on } U_2 = \{y \neq 0\}$$

& $$[(x,y,z)] \longmapsto (\omega_1, \omega_2) = (y/x, z/x) \text{ on } U_1 = \{x \neq 0\}.$$

Prove that $\exists$ a smooth vecter field on $\mathbb{RP}^2$

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which, on U_1, has the expression

$$\omega_1 \frac{\partial}{\partial \omega_1} - \omega_2 \frac{\partial}{\partial \omega_2}.$$

Write the expression for this vecter field in the other two charts.

3. M, N be smooth, $f: M \to N$ smooth. Suppose M is compact & N connected. If f is injective and $Df(p)$ is an isomorphism: $T_pM \to T_pN$ for all p, prove that f is a diffeomorphism.