Homework-7

1. Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be C^1. Compute the pullback $f^*(dx_1 \wedge \cdots \wedge dx_n)$; $x_1, \ldots, x_n$ denote the usual coordinates on $\mathbb{R}^n$.

2. The n-form $dx_1 \wedge \cdots \wedge dy_n$ on $\mathbb{R}^n$ is called the <u>volume</u> <u>form</u> on $\mathbb{R}^n$. Let $\omega = dx_1 \wedge \cdots \wedge dx_n$ & $f : \mathbb{R}^n \to \mathbb{R}^n$ smooth, $x \in \mathbb{R}^n$. Compute
$$f^*(\omega)(x) \in \Lambda^n (T_x \mathbb{R}^n)^* \equiv \Lambda^n T_x \mathbb{R}^n \equiv \Lambda^n \mathbb{R}^n.$$

3. Let g be an orientation preserving diffeom. $: U \xrightarrow{\sim} V$, both open subsets of $\mathbb{R}^n$. Let α be a compactly supported n-form on V. Show that
$$\int_U g^*(\alpha) = \int_V \alpha.$$

4. What can you say about the groups $H^r_{dR}(\mathbb{R}^n)$ for $r > n$?

5. Generalize your answer for 4!

More problems may be posted next week!