Homework-8

1. Let $F := (F_1, \cdots, F_m) : \mathbb{R}^n \to \mathbb{R}^m$ be a smooth function. Let $\{x_i\}$ be the coordinates on $\mathbb{R}^n$ & $\{y_j\}$ the coordinates in $\mathbb{R}^m$. Compute $F^* dy_i$, $1 \leq i \leq m$.
2. Let $f, g \in C^\infty(M)$. Prove that $d(fg) = f\,dg + g\,df$.
3. Prove that Lie groups are orientable.
4. Let $\dim M = m$, then $H^r_{dR}(M) = 0$, $r \geq m+1$.
5. Let $\varphi_1, \cdots, \varphi_p$ be 1-forms, $X_1, \cdots, X_p$ vector fields then $(\varphi_1 \wedge \cdots \wedge \varphi_p)(X_1, \cdots, X_p) = \sum_{\sigma \in S_p} \operatorname{sign}(\sigma)\, \varphi_1(X_{\sigma(1)}) \cdots \varphi_n(X_{\sigma(n)})$
6. Let ω be a differential form on $\mathbb{R}^n$. Is it true that $\omega \wedge \omega = 0$?
7. Consider $\varphi : \mathbb{R} \to \mathbb{R}^2 \setminus \{(0,0)\}$, $\varphi(\theta) = (\cos\theta, \sin\theta)$. Let ω be the differential 1-form on $\mathbb{R}^2 \setminus \{(0,0)\}$,
$$\omega = \frac{-y}{x^2+y^2}\,dx + \frac{x}{x^2+y^2}\,dy.$$
Compute the pull back $\varphi^* \omega \in D^1(\mathbb{R})$.
8. Consider $\mathbb{R}^2$ with polar coordinates (r, θ) and $\omega = (r \sin\theta)\,dr \in D^1(\mathbb{R}^2)$. Compute $d\omega$.
9. Let α, β & γ be differential forms with $d\alpha = 0 = d\beta = d\gamma$. What can you say about $d(\alpha \wedge \beta \wedge \gamma)$?
10. Let ω be as in (7). Show that ω is <u>not</u> exact.