QUIZ-4

1. Let V be a finite dimensional vector space over $\mathbb{R}$ and let $B: V \times V \to \mathbb{R}$ be a non-degenerate bilinear form on V, i.e. B is bilinear with $B(v, v') = 0 \ \forall\, v' \Rightarrow v = 0$ & $B(v, v') = 0 \ \forall\, v \Rightarrow v' = 0$. Let
$$H = \{A \in GL(V) \mid B(Ax, Ay) = B(x, y) \ \forall\, x, y \in V\}.$$
Show that H is a Lie subgroup of $GL(V)$ with
$$\mathfrak{h} = \{A \in \mathfrak{gl}(V) \mid B(Ax, y) + B(x, Ay) = 0 \ \forall\, x, y \in V\}$$
as its Lie algebra. (3)

2. Let G, H be Lie groups, $\mathfrak{g}, \mathfrak{h}$ the corresp. Lie algebras. Let $f: G \to H$ be a Lie group homomorphism. Show that $\mathrm{Ker}(f)$ is a normal Lie subgroup of G and
$$\mathrm{Lie}(\mathrm{Ker}\, f) = \ker(Df(e)).$$
(3).