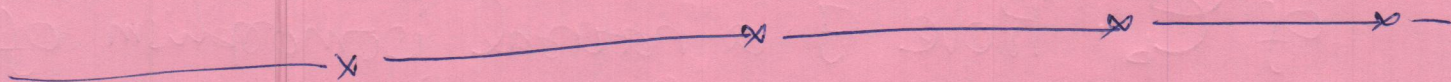


Quiz - 1

1. Let M, N be smooth manifolds and $f: M \rightarrow N$ be a smooth map. Prove that f is continuous. (3)

2. Let M, N be smooth manifolds, $f: M \rightarrow N$ be a constant map. Does this imply $Df(p): T_p M \rightarrow T_{f(p)} N$ is zero? Explain. (3)



QUIZ-2

1. Can we have a C^∞ 1-1 map: $\mathbb{R}^2 \rightarrow \mathbb{R}$?
Explain. (3)

2. Let G be a Lie group, $\mathfrak{g} = \text{Lie } G$,
 $X \in \mathfrak{g}$. Let $\text{Exp}_X : G \rightarrow G$ denote
the exponential map corresp. to X .
Prove that $\text{Exp}_X(g) = L_g(\exp(x)) \forall g \in G$,
Where $\exp(x) = \text{Exp}_X(e)$. Deduce that
the flow of X is given by
$$F_X(t, g) = g \exp(tx), t \in \mathbb{R}, g \in G.$$

(3).

QUIZ-4

1. Let V be a finite dimensional vector space over $\mathbb{R}$ and let $B: V \times V \rightarrow \mathbb{R}$ be a non-degenerate bilinear form on V , i.e. B is bilinear with $B(v, v') = 0 \ \forall v' \Rightarrow v = 0$ & $B(v, v') = 0 \ \forall v \Rightarrow v' = 0$. Let $H = \{A \in GL(V) \mid B(Ax, Ay) = B(x, y) \ \forall x, y \in V\}$.

Show that H is a Lie subgroup of $GL(V)$

with $\mathfrak{h} = \{A \in \mathfrak{gl}(V) \mid B(Ax, y) + B(x, Ay) = 0 \ \forall x, y \in V\}$ as its Lie algebra. (3)

2. Let G, H be Lie groups, $\mathfrak{g}, \mathfrak{h}$ the corresp. Lie algebras. Let $f: G \rightarrow H$ be a Lie group homomorphism. Show that $\text{Ker}(f)$ is a normal Lie subgroup of G and

$$\text{Lie}(\text{Ker } f) = \text{ker}(Df(e)). \quad (3).$$

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QUIZ-4

Note: This is the last quiz in the course.

1. Let $i: S^1 \hookrightarrow \mathbb{R}^2 \setminus \{(0,0)\}$ be the inclusion map and $\omega = \frac{-ydx + xdy}{x^2 + y^2} \in \mathcal{D}^1(\mathbb{R}^2 \setminus \{(0,0)\})$.

Show that

$$\int_{S^1} i^* \omega = 2\pi \quad (3)$$

2. Show that the differential form ω in Problem (1) is closed but not exact.

(3)

