Quiz-1

1. Let M, N be smooth manifolds and $f: M \to N$ be a smooth map. Prove that f is continuous. (3)

2. Let M, N be smooth manifolds, $f: M \to N$ be a constant map. Does this imply $Df(p): T_pM \to T_{f(p)}N$ is zero? Explain. (3)

QUIZ-2

1. Can we have a C^∞ 1-1 map: $\mathbb{R}^2 \to \mathbb{R}$? Explain. (3)

2. Let G be a Lie group, $\mathfrak{g} = \mathrm{Lie}\, G$, $X \in \mathfrak{g}$. Let $\mathrm{Exp}_X : G \to G$ denote the exponential map corresp. to X. Prove that $\mathrm{Exp}_X(g) = L_g(\exp(X))$ $\forall g \in G$, where $\exp(X) = \mathrm{Exp}_X(e)$. Deduce that the flow of X is given by

$$F_X(t, g) = g \exp(tX), \quad t \in \mathbb{R}, \; g \in G.$$

(3).

QUIZ-4

1. Let V be a finite dimensional vector space over $\mathbb{R}$ and let $B : V \times V \to \mathbb{R}$ be a non-degenerate bilinear form on V, i.e. B is bilinear with $B(v, v') = 0 \ \forall v' \Rightarrow v = 0$ & $B(v, v') = 0 \ \forall v \Rightarrow v' = 0$. Let $H = \{A \in GL(V) \mid B(Ax, Ay) = B(x, y) \ \forall x, y \in V\}$.

   Show that H is a Lie subgroup of $GL(V)$ with $\mathfrak{h} = \{A \in \mathfrak{gl}(V) \mid B(Ax, y) + B(x, Ay) = 0 \ \forall x, y \in V\}$ as its Lie algebra. (3)

2. Let G, H be Lie groups, $\mathfrak{g}, \mathfrak{h}$ the corresp. Lie algebras. Let $f : G \to H$ be a Lie group homomorphism. Show that $\mathrm{Ker}(f)$ is a normal Lie subgroup of G and

   $$\mathrm{Lie}(\mathrm{Ker}\, f) = \ker(Df(e)).$$ (3).

QUIZ-4

Note: This is the last quiz in the course.

1. Let $i : S^1 \hookrightarrow \mathbb{R}^2 \setminus \{(0,0)\}$ be the inclusion map and $\omega = \dfrac{-y\,dx + x\,dy}{x^2+y^2}$ $\in \mathcal{D}^1(\mathbb{R}^2 \setminus \{(0,0)\})$.

   Show that

$$\int_{S^1} i^*(\omega) = 2\pi$$

(3)

2. Show that the differential form ω in Problem (1) is closed but not exact.

(3)