

**In problems 1 to 7,  $C$  denotes a simple, rectifiable, closed contour in  $\mathbb{C}$ .**

**Q 0.** Note that  $\sqrt{z}$  is defined on the complement of  $(-\infty, 0]$  with the unique value where the real part is positive. This function is continuous (even holomorphic) on this complement. Describe/discuss/verify clearly what happens to the limits of the chosen values of  $\sqrt{z}$  when a point of the half-line (the ‘cut’) is approached from both sides. Similarly, on the complement of  $[0, \infty)$ , note that  $\sqrt{z}$  is defined with the unique value which has positive imaginary part. Discuss analogously what happens to the two limits when we similarly approach a point on the positive half-line from both sides.

**Q 1.** Determine all possible values of the integral  $\int_C \frac{dz}{z(z^2-1)}$  depending on the relative positions of  $-1, 0, 1$  with respect to the contour  $C$ , assuming that none of  $-1, 0, 1$  lie on  $C$ . If instead of  $-1, 0, 1$ , we choose  $n$  distinct points not lying on  $C$ , how many different values of the integral do we obtain?

**Q 2.** Compute  $\int_{|z-t|=t} \frac{zdz}{z^4-1}$  if  $t > 1$  is real.

**Q 3.** If the closed disc  $|z| \leq r$  lies inside  $C$ , evaluate  $\int_C \frac{e^z dz}{z^2+r^2}$ .

**Q 4.** If  $z_0$  lies inside  $C$ , find  $\int_C \frac{ze^z dz}{(z-z_0)^3}$ .

**Q 5.** If at least one of  $0, 1$  lies within  $C$ , compute  $\int_C \frac{e^z dz}{z(1-z)^3}$ .

**Q 6.** Assume that  $C$  contains 0 in its interior  $D$ , and let  $f$  be holomorphic in the domain  $D$ . If, for  $z \in C$ , and ANY of the possible values of  $\log(z)$ , prove that  $\int_C f'(z) \log(z) dz = 2i\pi(f(z_0) - f(0))$  where  $z_0$  is the starting point of the integration.

**Q 7.** Find the value of  $\int_C z^2 \text{Log}\left(\frac{z-1}{z+1}\right) dz$  if  $C$  is the circle  $|z-1| = 1$  starting from the point  $1+i$ .