

In problems 1 to 7, C denotes a simple, rectifiable, closed contour in \mathbb{C} .

Q 0. Note that \sqrt{z} is defined on the complement of $(-\infty, 0]$ with the unique value where the real part is positive. This function is continuous (even holomorphic) on this complement. Describe/discuss/verify clearly what happens to the limits of the chosen values of \sqrt{z} when a point of the half-line (the ‘cut’) is approached from both sides. Similarly, on the complement of $[0, \infty)$, note that \sqrt{z} is defined with the unique value which has positive imaginary part. Discuss analogously what happens to the two limits when we similarly approach a point on the positive half-line from both sides.

Q 1. Determine all possible values of the integral $\int_C \frac{dz}{z(z^2-1)}$ depending on the relative positions of $-1, 0, 1$ with respect to the contour C , assuming that none of $-1, 0, 1$ lie on C . If instead of $-1, 0, 1$, we choose n distinct points not lying on C , how many different values of the integral do we obtain?

Q 2. Compute $\int_{|z-t|=t} \frac{zdz}{z^4-1}$ if $t > 1$ is real.

Q 3. If the closed disc $|z| \leq r$ lies inside C , evaluate $\int_C \frac{e^z dz}{z^2+r^2}$.

Q 4. If z_0 lies inside C , find $\int_C \frac{ze^z dz}{(z-z_0)^3}$.

Q 5. If at least one of $0, 1$ lies within C , compute $\int_C \frac{e^z dz}{z(1-z)^3}$.

Q 6. Assume that C contains 0 in its interior D , and let f be holomorphic in the domain D . If, for $z \in C$, and ANY of the possible values of $\log(z)$, prove that $\int_C f'(z)\log(z)dz = 2i\pi(f(z_0) - f(0))$ where z_0 is the starting point of the integration.

Q 7. Find the value of $\int_C z^2 \text{Log}\left(\frac{z-1}{z+1}\right) dz$ if C is the circle $|z-1| = 1$ starting from the point $1+i$.