Percolation Theory

Homework 6

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$$\mathcal{L} = \left\{ A \in \mathcal{F} : \forall n \geq 1, \exists\ D_n \in \mathcal{C} \text{ such that } \mathbb{P}_p\left(A \Delta D_n\right) \leq \frac{1}{n} \right\}$$

Problem 1. Show that $\mathcal{L}$ is a π-system.

Solution. Clearly, $\Omega \in \mathcal{L}$, as Ω is a cylinder set. To show the closure under finite intersection, it is sufficient to show for two sets, as the former follows by induction. Let $A, B \in \mathcal{L}$. Then, there exist $D_n, E_n \in \mathcal{C}$ such that $\mathbb{P}_p\left(A \Delta D_n\right) \leq \frac{1}{2n}$ and $\mathbb{P}_p\left(B \Delta E_n\right) \leq \frac{1}{2n}$. Now, we have

$$\begin{aligned}
\mathbb{P}_p\left((A \cap B)\Delta(D_n \cap E_n)\right) &\leq \mathbb{P}_p\left((A \cap B)\Delta(D_n \cap B)\right) + \mathbb{P}_p\left((D_n \cap B)\Delta(D_n \cap E_n)\right) \\
&\leq \mathbb{P}_p\left(A \Delta D_n\right) + \mathbb{P}_p\left(B \Delta E_n\right) \\
&\leq \frac{1}{2n} + \frac{1}{2n} = \frac{1}{n}.
\end{aligned}$$

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