

INDIAN STATISTICAL INSTITUTE, KOLKATA

Assignment 8 , Second Semester 2024-25

Algebra , M. Math I

Date :

1. Let $k = \mathbb{Q}$ and $L = \mathbb{Q}(\sqrt{2})$. Find out $N_{L/k}(a + b\sqrt{2})$ and $tr(a + b\sqrt{2})$.
2. Let $k = \mathbb{Q}$ and $L = \mathbb{Q}(2^{1/3})$. Find out $N_{L/k}(a + b2^{1/3} + c4^{1/3})$ and $tr_{L/k}(a + b2^{1/3} + c4^{1/3})$.
3. Using norm show that $1 + 5.2^{1/3} - 4^{1/3}$ is not a perfect square in $\mathbb{Q}(2^{1/3})$.
4. Find out the discriminant of the polynomial $x^3 - 2$ over \mathbb{Q} .
5. Let L/k be a separable extension of degree n and let $L \subset N$ subfield such that N/k is normal. Moreover, let $\{\sigma_1, \dots, \sigma_n\}$ be n -distinct k homomorphism $L \rightarrow N$ and $\alpha_1, \dots, \alpha_n \in L$. Show that

$$\det((\sigma_i(\alpha_j))_{1 \leq i \leq n, 1 \leq j \leq n})^2 = \det((tr_{L/k}(\alpha_i \cdot \alpha_j))_{1 \leq i \leq n, 1 \leq j \leq n}).$$

(Hint : If M be the matrix $(\sigma_i(\alpha_j))_{1 \leq i \leq n, 1 \leq j \leq n}$, then find out $M \cdot M^t$.)