

INDIAN STATISTICAL INSTITUTE, KOLKATA

Assignment 8 , Second Semester 2024-25

Algebra , M. Math I

Date :

1. Let  $k = \mathbb{Q}$  and  $L = \mathbb{Q}(\sqrt{2})$ . Find out  $N_{L/k}(a + b\sqrt{2})$  and  $tr(a + b\sqrt{2})$ .
2. Let  $k = \mathbb{Q}$  and  $L = \mathbb{Q}(2^{1/3})$ . Find out  $N_{L/k}(a + b2^{1/3} + c4^{1/3})$  and  $tr_{L/k}(a + b2^{1/3} + c4^{1/3})$ .
3. Using norm show that  $1 + 5.2^{1/3} - 4^{1/3}$  is not a perfect square in  $\mathbb{Q}(2^{1/3})$ .
4. Find out the discriminant of the polynomial  $x^3 - 2$  over  $\mathbb{Q}$ .
5. Let  $L/k$  be a separable extension of degree  $n$  and let  $L \subset N$  subfield such that  $N/k$  is normal. Moreover, let  $\{\sigma_1, \dots, \sigma_n\}$  be  $n$ -distinct  $k$  homomorphism  $L \rightarrow N$  and  $\alpha_1, \dots, \alpha_n \in L$ . Show that

$$\det((\sigma_i(\alpha_j))_{1 \leq i \leq n, 1 \leq j \leq n})^2 = \det((tr_{L/k}(\alpha_i \cdot \alpha_j))_{1 \leq i \leq n, 1 \leq j \leq n}).$$

(Hint : If  $M$  be the matrix  $(\sigma_i(\alpha_j))_{1 \leq i \leq n, 1 \leq j \leq n}$ , then find out  $M.M^t$ .)