INDIAN STATISTICAL INSTITUTE

Semester	:	Second Semester 2024-25
Course	:	M.Math 1st Year
Subject	:	Functional Analysis
Name	:	
Roll No	:	

Date : 8th April 2025 Marks : Duration: 30 minutes

1. (**For roll No: 2308, 2304, 41, 05, 35, 39, 04, 34**) Let $A \in M_n(\mathbb{C})$ satisfy $\lim_{\lambda \to 0}(A - \lambda I)^{-1} = B$. Show that $0 \notin \sigma(A)$. **[5]**

2. (**For roll No: 03, 09, 30, 33, 06, 36, 42**) Suppose $T \in \mathcal{B}(\mathcal{H})$ satisfies $TT^*TT^* = TT^*$. Then show that $TT^*T = T$. **[5]**

3. (**For roll No: 28, 43, 16, 22, 25, 12**) Suppose $T \in \mathcal{B}(\mathcal{H})$ satisfies $T^*TT^*T = T^*T$. Then show that $T^*TT^* = T^*$. **[5]**

4. (**For roll No: 02, 37, 18, 31, 29, 08**) Let $\mathcal{H}$ be a Hilbert space, $u, v \in \mathcal{H}$ and $A \in \mathcal{B}(\mathcal{H})$. Suppose there exists $T > 0$ satisfying $\langle v, (\lambda I + A)^{-1} u \rangle = 0, \forall \lambda$ with $|\lambda| > T$. Show that $\langle v, u \rangle = 0$. If it helps you can assume $\mathcal{H}$ to be finite dimensional. **[5]**

5. (**For roll No: 17, 20, 24, 13, 23**) Let $T \in \mathcal{B}(\mathcal{H})$ be a finite rank operator. Show that $\dim \ker(I - T) = \dim \ker(I - T^*)$. **[5]**

6. (**For roll No: 11,19,21,26,27,32,28**) Let $(\Omega, \mathfrak{S}, P)$ be a probability space and $f : (\Omega, \mathfrak{S}) \to (\mathbb{R}, \mathcal{B}_{\mathbb{R}})$ be a bounded measurable function. Consider $M_f \in \mathcal{B}(L^2(P))$ given by $M_f(\xi) = f \cdot \xi$. Show that $\sigma(M_f) \subseteq \overline{\{\langle \xi, M_f(\xi) \rangle : \|\xi\| = 1\}}$. **[5]**