Indian Statistical Institute

Course Name: M. Math, 1^{st} year
Subject Name : Topology II
Exam name : Final Part I
Maximum Marks: 20, Duration: 40 min
Date: 23.07.2021, 2:00 PM - 2:40 PM

- You may use any results proved in class. Any other results (including those in homework problem sets) require proof. You are required to provide proper mathematical reasons for your answers.

1. Prove tbat $\mathbb{R}P^{2n}$ is not homotopy equivalent to $\mathbb{R}P^{2n-1} \vee S^{2n}$. 6
2. Compute the homology of $\mathbb{R}P^{n+k}/\mathbb{R}P^n$ (with both $\mathbb{Z}$ and $\mathbb{Z}/2$-coefficients). 6
3. Is there a CW complex structure on $\mathbb{R}P^2$ with 5 0-cells, 5 1-cells, and 5 2-cells ? 8

Course Name: M. Math, 1^{st} year
Subject Name : Topology II
Exam name : Final Part II
Maximum Marks: 20, Duration: 40 min
Date: 23.07.2021, 3:00 PM - 3:40 PM

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1. Let X be a simply connected CW complex. Prove that any map from X to $(S^1)^k$ induces the 0 map on reduced homology. 6
2. Let p, q be two points in S^2, $p \neq q$. Suppose $r \in S^2$ is different from p and q. Find $\pi_1(S^2 - p - q, r)$. 6
3. Let X be the mapping cone of $\phi_k : S^1 \to S^1$ given by $\phi_k(z) = z^k$. Find $\pi_1 X$. 8

Course Name: M. Math, 1^{st} year
Subject Name : Topology II
Exam name : Final Part III
Maximum Marks: 20, Duration: 40 min
Date: 23.07.2021, 4:00 PM - 4:40 PM

- You may use any results proved in class. Any other results (including those in homework problem sets) require proof. You are required to provide proper mathematical reasons for your answers.

1. Calculate $H_*(X \times \Sigma_g)$ if X is a CW complex. 6

2. Construct a surjective map $S^n \to S^n$ of degree 0 for each $n \geq 1$. 6

3. Let X be a CW complex with 1 cell in degree 0, r cells in degree 2 (where r is some integer ≥ 0), 1 cell in degree 4, and no cells in any other degrees. Find the homology groups of X. 8